$$V_{LT} = \frac{100}{50100} \times (-14) = -28\text{ mV}$$

Example 5.3

A Schmitt trigger with the upper threshold level $V_{UT} = 0$ V and hysteresis width $V_H = 0.2$ V converts a 1 kHz sine wave of amplitude $4V_{pp}$ into a square wave. Calculate the time duration of the negative and positive portion of the output waveform.

Solution

$V_{UT} = 0$

$V_H = V_{UT} - V_{LT} = 0.2$ V

So, $V_{LT} = -0.2$ V

In Fig. 5.9, the angle θ can be calculated as

$$-0.2 = V_m \sin(\pi + \theta) = -V_m \sin\theta = -2\sin\theta$$

$$\theta = \text{arc sin } 0.1 = 0.1 \text{ radian}$$

The period, $T = 1/f = 1/1000 = 1$ ms

$$wT_\theta = 2\pi\ (1000)\ T_\theta = 0.1$$

$$T_\theta = (0.1/2\pi) \text{ ms} = 0.016 \text{ ms}$$

So, $T_1 = T/2 + T_\theta = 0.516$ ms

and $T_2 = T/2 - T_\theta = 0.484$ ms

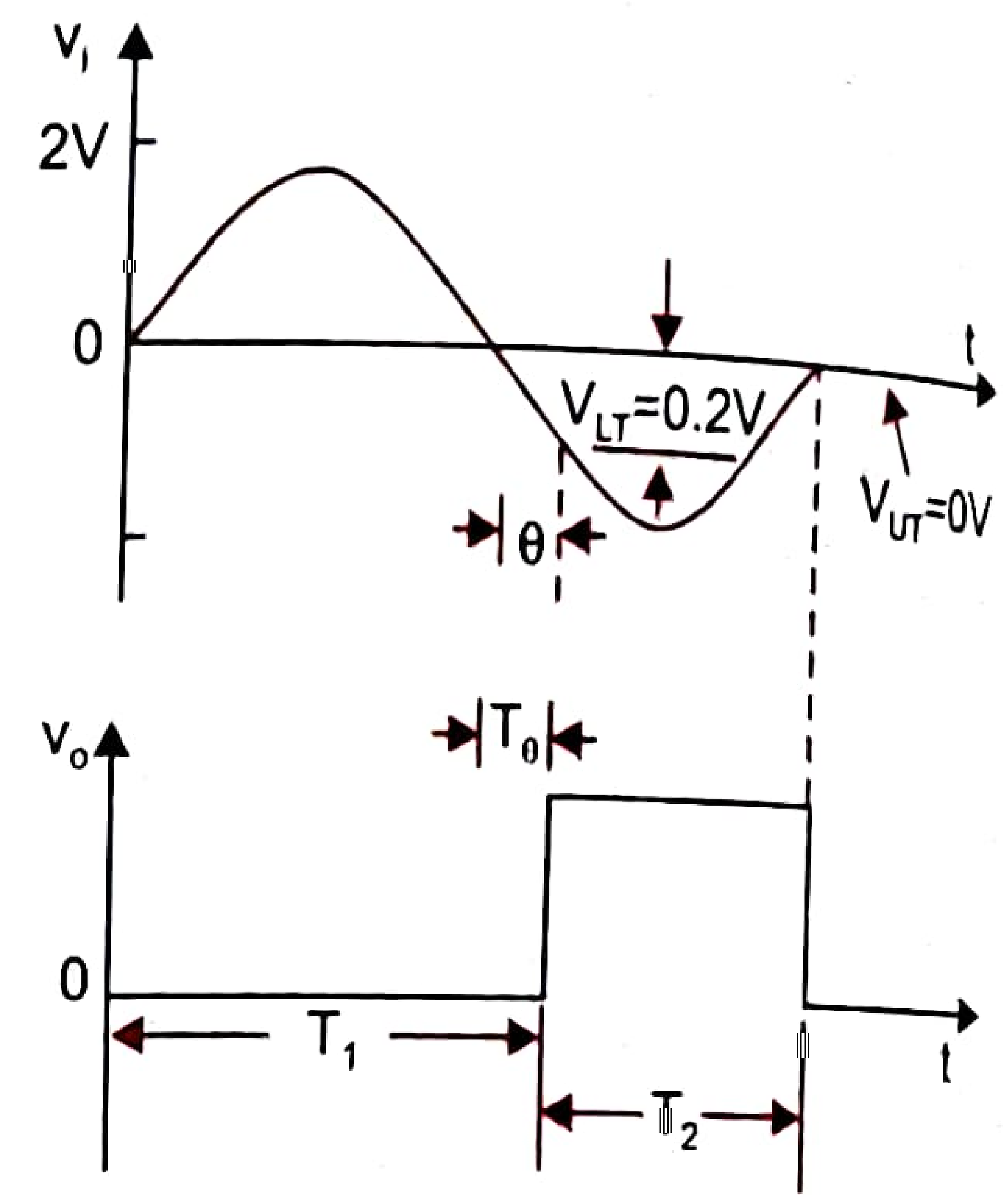

Fig. 5.9 Circuit for Example 5.3

5.4 SQUARE WAVE GENERATOR (ASTABLE MULTIVIBRATOR)

A simple op-amp square wave generator is shown in Fig. 5.10 (a). Also called a ***free running oscillator,*** the principle of generation of square wave output is to force an op-amp to operate in the saturation region. In Fig. 5.10 (a) fraction $\beta = R_2/(R_1 + R_2)$ of the output is fed back to the (+) input terminal. Thus the reference voltage V_{ref} is βv_o and may take values as

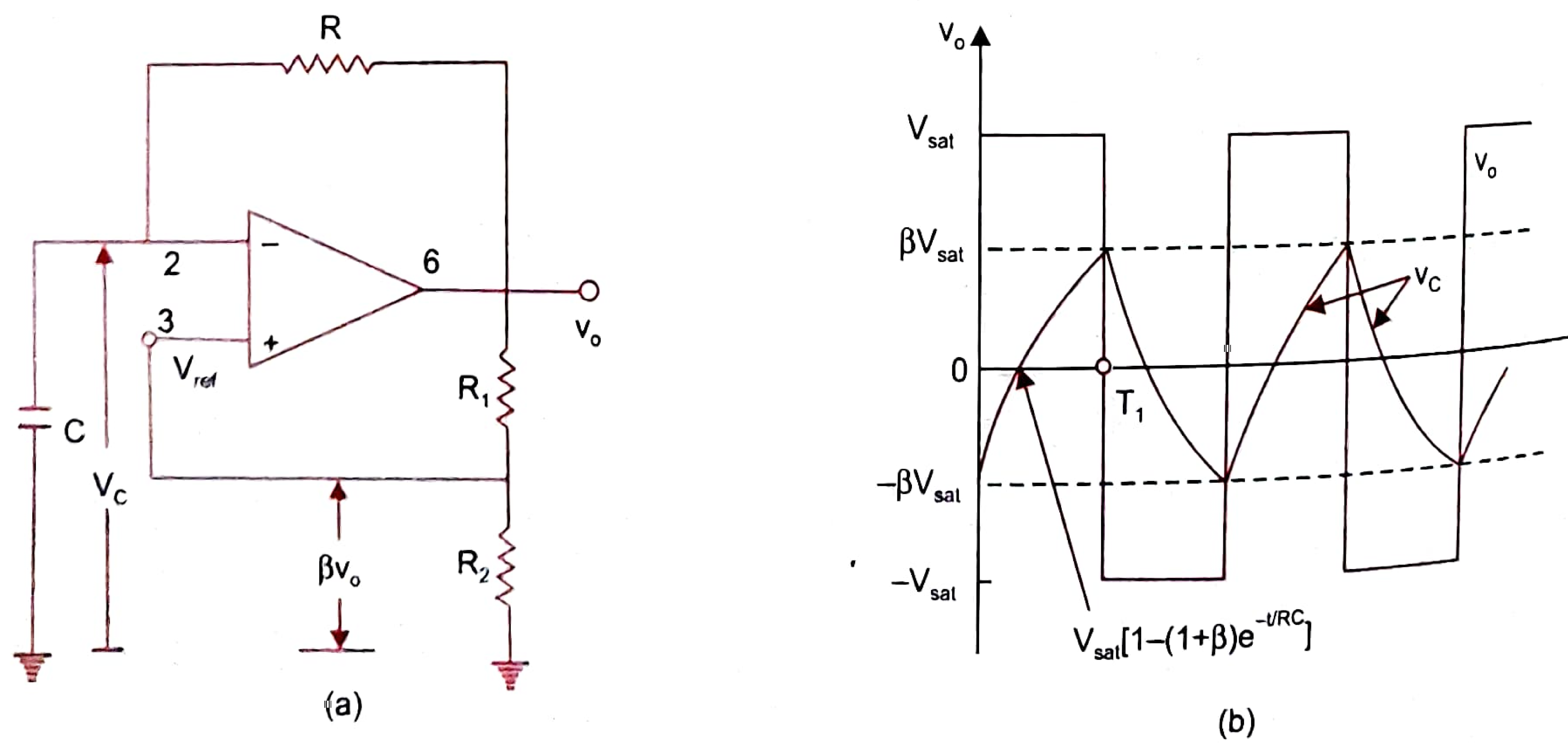

Fig. 5.10 (a) Simple op-amp square wave generator (b) Waveforms

$+\beta V_{sat}$ or $-\beta V_{sat}$. The output is also fed back to the (–) input terminal after integrating by means of a low-pass RC combination. Whenever input at the (–) input terminal just exceeds V_{ref} switching takes place resulting in a square wave output. In astable multivibrator, both the states are quasi stable.

Consider an instant of time when the output is at $+V_{sat}$. The capacitor now starts charging towards $+V_{sat}$ through resistance R, as shown in Fig. 5.10 (b). The voltage at the (+) input terminal is held at $+\beta V_{sat}$ by R_1 and R_2 combination. This condition continues as the charge on C rises, until it has just exceeded $+\beta V_{sat}$, the reference voltage. When the voltage at the (–) input terminal becomes just greater than this reference voltage, the output is driven to $-V_{sat}$. At this instant, the voltage on the capacitor is $+\beta V_{sat}$. It begins to discharge through R, that is, charges toward $-V_{sat}$. When the output voltage switches to $-V_{sat}$, the capacitor charges more and more negatively until its voltage just exceeds $-\beta V_{sat}$. The output switches back to $+V_{sat}$. The cycle repeats itself as shown in Fig. 5.10 (b).

The frequency is determined by the time it takes the capacitor to charge from $-\beta V_{sat}$ to $+\beta V_{sat}$ and vice versa. The voltage across the capacitor as a function of time is given by,

$$v_c(t) = V_f + (V_i - V_f)e^{-t/RC} \tag{5.4}$$

where, the final value, $V_f = +V_{sat}$

and the initial value, $V_i = -\beta V_{sat}$

Therefore,

$$v_c(t) = V_{sat} + (-\beta V_{sat} - V_{sat})\, e^{-t/RC}$$

or

$$v_c(t) = V_{sat} - V_{sat}\,(1 + \beta)\, e^{-t/RC} \tag{5.5}$$

At $t = T_1$, voltage across the capacitor reaches βV_{sat} and switching takes place. Therefore,

$$v_c(T_1) = \beta V_{sat} = V_{sat} - V_{sat}\,(1+\beta)e^{-T_1/RC} \tag{5.6}$$

After algebraic manipulation, we get,

$$T_1 = RC \ln \frac{1+\beta}{1-\beta} \tag{5.7}$$

This give only one half of the period.

Total time period

$$T = 2T_1 = 2RC \ln \frac{1+\beta}{1-\beta} \tag{5.8}$$

and the output wave form is symmetrical.

If $R_1 = R_2$, then $\beta = 0.5$, and $T = 2RC \ln 3$. And for $R_1 = 1.16R_2$, it can be seen that $T = 2\,RC$

or

$$f_0 = \frac{1}{2RC}$$

The output swings from $+V_{sat}$ to $-V_{sat}$, so,

$$v_o \text{ peak-to-peak} = 2\,V_{sat} \tag{5.9}$$

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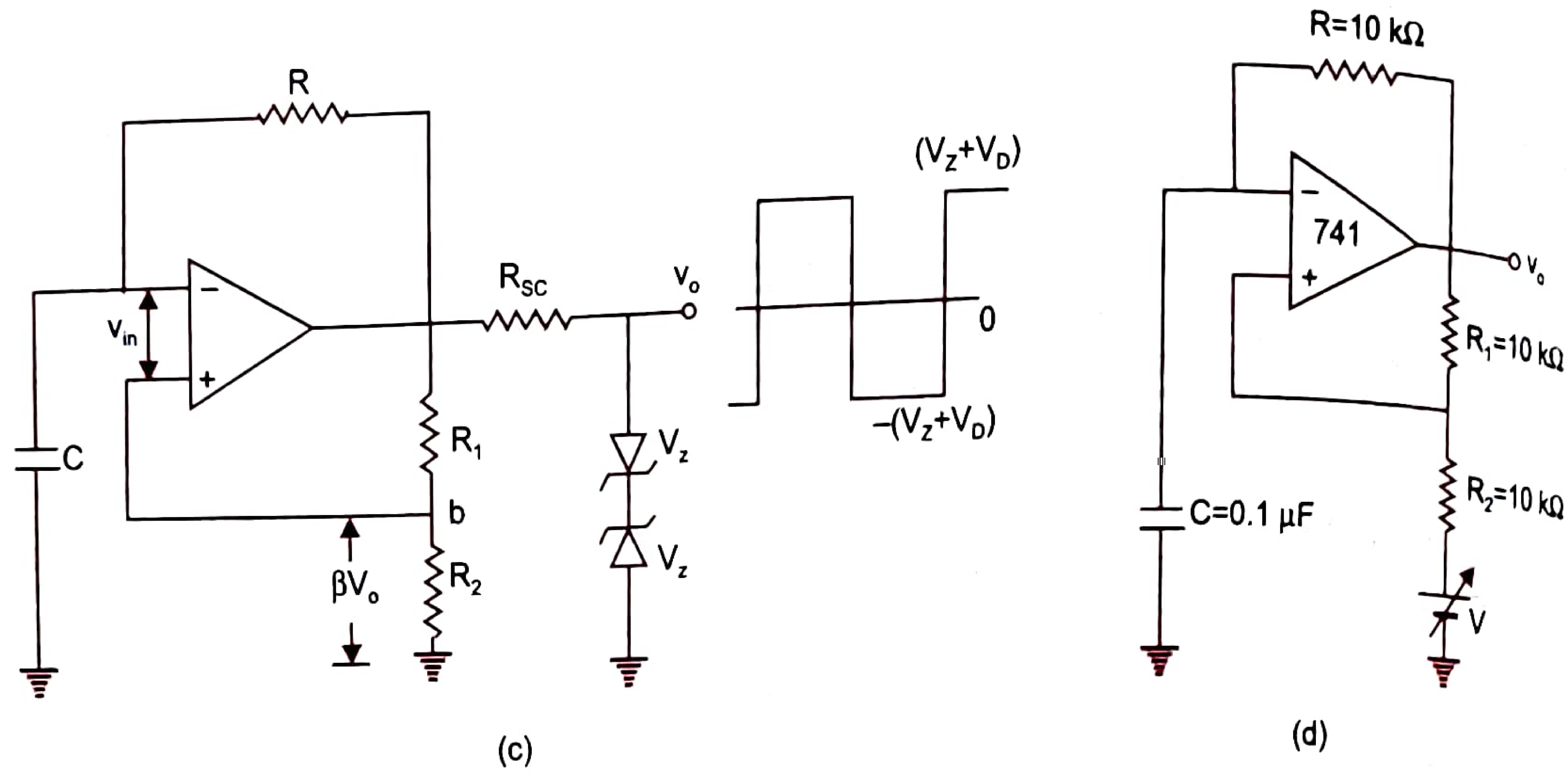


Fig. 5.10 (c) Use of back to back zener diodes. (d) Asymmetric square wave generator

The peak to peak output amplitude can be varied by varying the power supply voltage. However, a better technique is to use back to back zener diodes as shown in Fig. 5.10 (c). The output voltage is regulated to ± ($V_Z + V_D$) by the zener diodes.

$$v_o \text{ peak-to-peak} = 2\,(V_Z + V_D) \tag{5.10}$$

Resistor R_{sc} limits the currents drawn from the op-amp to,

$$I_{sc} = \frac{V_{sat} - V_Z}{R_{sc}} \tag{5.11}$$

This circuit works reasonably well at audio frequencies. At higher frequencies, however, slew-rate of the op-amp limits the slope of the output square wave.

If an ***asymmetric square wave*** is desired, then zener diodes with different break down voltages V_{Z1} and V_{Z2} may be used. Then the output is either V_{o1} or V_{o2}, where $V_{o1} = V_{Z1} + V_D$ and $V_{o2} = V_{Z2} + V_D$. It can be easily shown that the positive section is given by,

$$T_1 = RC \ln \frac{1 + \beta V_{o2}/V_{o1}}{1 - \beta} \tag{5.12}$$

The duration of negative section T_2 will be the same as given by Eq. (5.12) with V_{o1} and V_{o2} interchanged.

An alternative method to get asymmetric square wave output is to add a dc voltage source V in series R_2 as shown in Fig. 5.10 (d). Now the capacitor C swings between the voltage levels ($\beta V_{sat} + V$) and ($-\beta V_{sat} + V$). If the voltage source V is made variable, voltage to frequency conversion can be achieved though the variation will not be linear.

5.5 MONOSTABLE MULTIVIBRATOR

Monostable multivibrator has one stable state and the other is quasi stable state. The circuit is useful for generating single output pulse of adjustable time duration in response to a triggering signal. The width of the output pulse depends only on external components connected to the op-amp. The circuit shown in Fig. 5.11(a) is a modified form of the astable multivibrator.

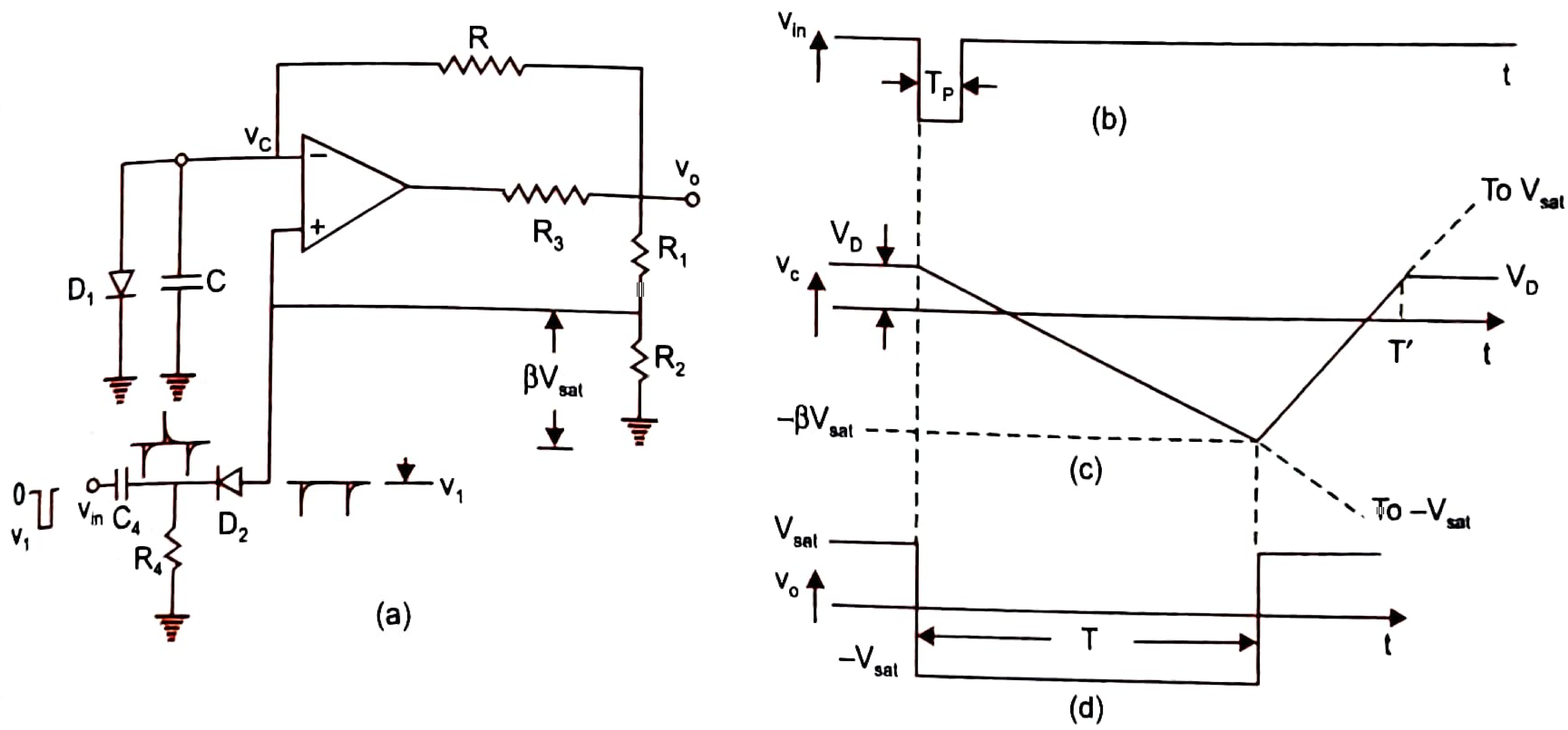


Fig. 5.11 (a) Monostable multivibrator, (b) Negative going triggering signal, (c) Capacitor waveform, (d) Output voltage waveform

A diode D_1 clamps the capacitor voltage to 0.7 V when the output is at $+V_{sat}$. A negative going pulse signal of magnitude V_1 passing through the differentiator R_4C_4 and diode D_2 produces a negative going triggering impulse and is applied to the (+) input terminal.

To analyse the circuit, let us assume that in the stable state, the output v_o is at $+V_{sat}$. The diode D_1 conducts and v_c the voltage across the capacitor C gets clamped to +0.7 V. The voltage at the (+) input terminal through R_1R_2 potentiometric divider is $+\beta V_{sat}$. Now, if a negative trigger of magnitude V_1 is applied to the (+) input terminal so that the effective signal at this terminal is less than 0.7 V, i.e. ($[\beta V_{sat} + (-V_1)] < 0.7$ V), the output of the op-amp will switch from $+V_{sat}$ to $-V_{sat}$. The diode will now get reverse biased and the capacitor starts charging exponentially to $-V_{sat}$ through the resistance R. The voltage at the (+) input terminal is now $-\beta V_{sat}$. When the capacitor voltage v_c becomes just slightly more negative than $-\beta V_{sat}$, the output of the op-amp switches back to $+V_{sat}$. The capacitor C now starts charging to $+V_{sat}$ through R until v_c is 0.7V as capacitor C gets clamped to the voltage. Various waveforms are shown in Fig. 5.11 (b, c, d).

The pulse width T of monostable multivibrator is calculated as follows:

The general solution for a single time constant low pass RC circuit with V_i and V_f as initial and final values is,

$$v_o = V_f + (V_i - V_f)e^{-t/RC} \tag{5.13}$$

For the circuit, $V_f = -V_{sat}$ and $V_i = V_D$ (diode forward voltage).

The output v_c is,

$$v_c = -V_{sat} + (V_D + V_{sat})\, e^{-t/RC} \tag{5.14}$$

at $t = T$,

$$v_c = -\beta V_{sat} \tag{5.15}$$

Therefore,

$$-\beta V_{sat} = -V_{sat} + (V_D + V_{sat})e^{-T/RC}$$

After simplification, pulse width T is obtained as

$$T = RC \ln \frac{(1 + V_D/V_{sat})}{1 - \beta} \tag{5.16}$$

where $\beta = R_2/(R_1 + R_2)$

If, $V_{sat} >> V_D$ and $R_1 = R_2$ so that $\beta = 0.5$, then

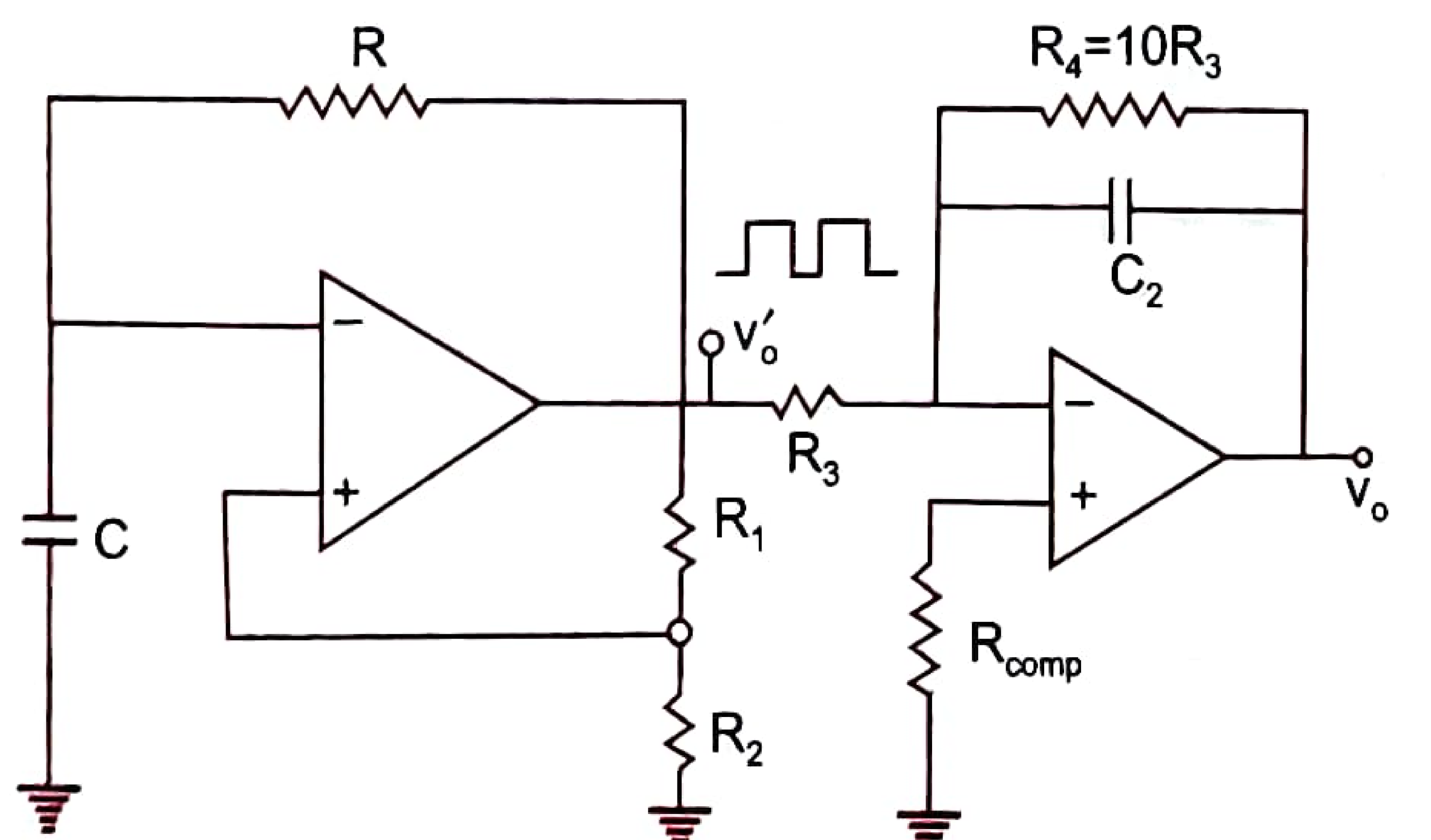
$$T = 0.69\ RC \tag{5.17}$$

For monostable operation, the trigger pulse width T_p should be much less than T, the pulse width of the monostable multivibrator. The diode D_2 is used to avoid malfunctioning by blocking the positive noise spikes that may be present at the differentiated trigger input.

It may be noted from Fig. 5.11 (b) that capacitor voltage v_c reaches its quiescent value V_D at $T' > T$. Therefore, it is essential that a recovery time $T' - T$ be allowed to elapse before the next triggering signal is applied. The circuit of Fig. 5.11 (a) can be modified to achieve voltage to time delay conversion as in the case of square wave generator. The monostable multivibrator circuit is also referred to as time delay circuit as it generates a fast transition at a predetermined time T after the application of input trigger. It is also called a gating circuit as it generates a rectangular waveform at a definite time and thus could be used to gate parts of a system.

5.6 TRIANGULAR WAVE GENERATOR

A triangular wave can be simply obtained by integrating a square wave as shown in Fig. 5.12 (a). It is obvious that the frequency of the square wave and triangular wave is the same as shown in Fig. 5.12 (b). Although the amplitude of the square wave is constant at $\pm V_{sat}$, the amplitude of the triangular wave will decrease as the frequency increases. This is because the reactance of the capacitor C_2 in the feedback circuit decreases at high frequencies. A resistance R_4 is connected across C_2 to avoid the saturation problem at low frequencies as in the case of practical integrator.



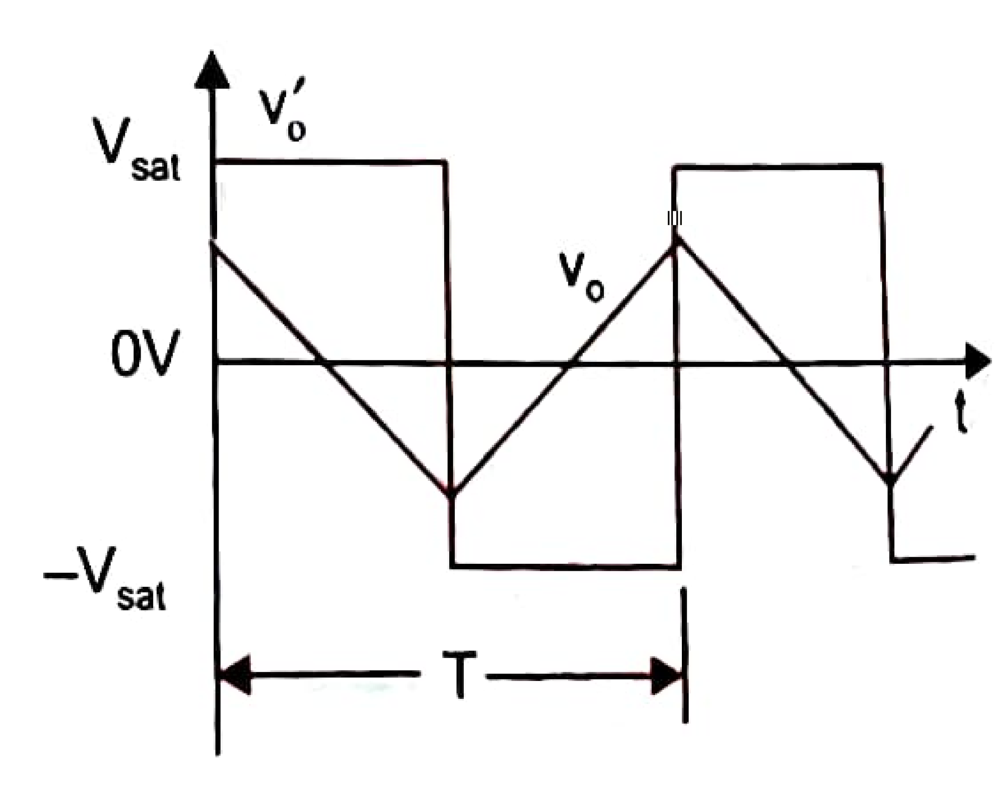


Fig. 5.12 (a) Triangular waveform generator, (b) Output waveform

Another triangular wave generator using lesser number of components is shown in Fig. 5.13 (a). It basically consists of a two level comparator followed by an integrator. The output of the comparator A_1 is a square wave of amplitude $\pm V_{sat}$ and is applied to the (–) input terminal of the integrator A_2 producing a triangular wave. This triangular wave is fed back as input to the comparator A_1 through a voltage divider R_2R_3.

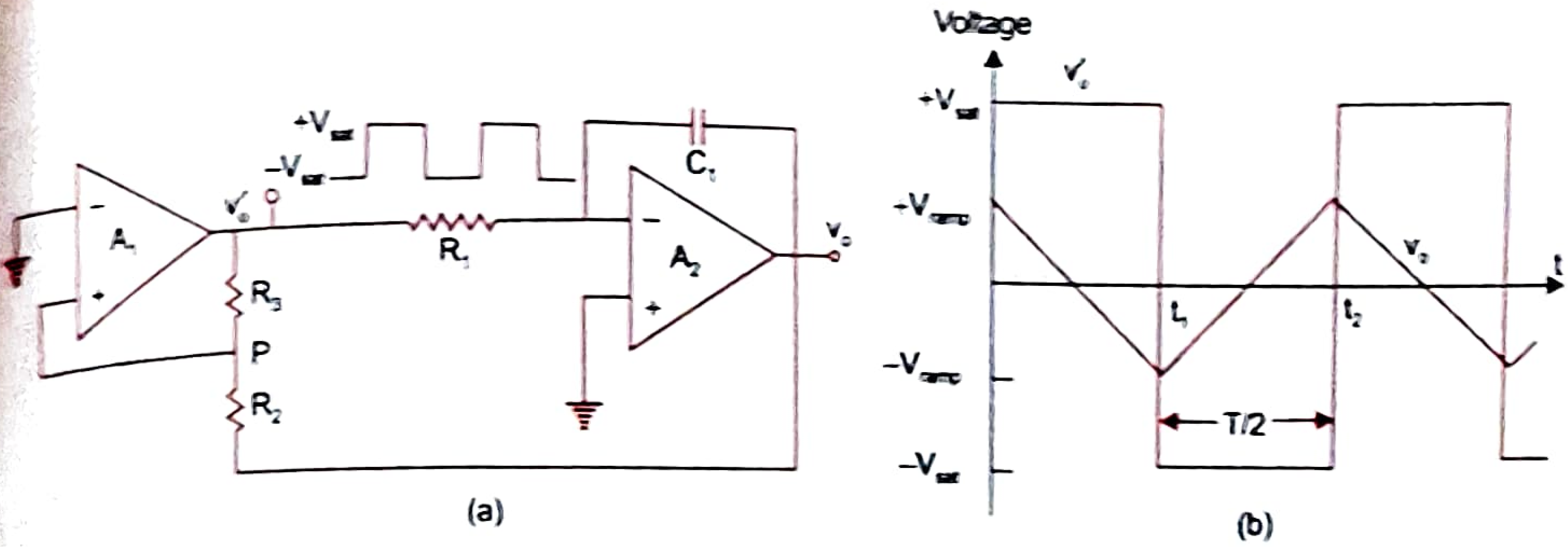


Fig. 5.13 (a) Triangular waveform generator using lesser components, (b) Waveforms

Initially, let us consider that the output of comparator A_1 is at $+V_{sat}$. The output of the integrator A_2 will be a negative going ramp as shown in Fig. 5.13 (b). Thus one end of the voltage divider R_2R_3 is at a voltage $+V_{sat}$ and the other at the negative going ramp of A_2. At a time $t = t_1$, when the negative going ramp attains a value of $-V_{ramp}$, the effective voltage at point P becomes slightly less than 0 V. This switches the output of A_1 from positive saturation to negative saturation level $-V_{sat}$. During the time when the output of A_1 is at $-V_{sat}$, the output of A_2 increases in the positive direction. And at the instant $t = t_2$, the voltage at point P becomes just above 0 V, thereby switching the output of A_1 from $-V_{sat}$ to $+V_{sat}$. The cycle repeats and generates a triangular waveform. It can be seen that the frequency of the square wave and triangular wave will be the same. However, the amplitude of the triangular wave depends upon the RC value of the integrator A_2 and the output voltage level of A_1. The output voltage of A_1 can be set to desired level by using appropriate zener diodes. The frequency of the triangular waveform can be calculated as follows:

The effective voltage at point P during the time when output of A_1 is at $+V_{sat}$ level is given by,

$$-V_{ramp} + \frac{R_2}{R_2 + R_3}\left[+V_{sat} - (-V_{ramp})\right] \tag{5.18}$$

At $t = t_1$, the voltage at point P becomes equal to zero. Therefore, from Eq. (5.18),

$$-V_{ramp} = -\frac{R_2}{R_3}(+V_{sat}) \tag{5.19}$$

Similarly, at $t = t_2$, when the output of A_1 switches from $-V_{sat}$ to $+V_{sat}$,

$$V_{ramp} = \frac{-R_2}{R_3}(-V_{sat}) = \frac{R_2}{R_3}(V_{sat}) \tag{5.20}$$

Therefore, peak to peak amplitude of the triangular wave is,

$$v_o\,(pp) = +V_{ramp} - (-V_{ramp}) = 2\frac{R_2}{R_3}V_{sat} \tag{5.21}$$

The output switches from $-V_{ramp}$ to $+V_{ramp}$ in half the time period $T/2$. Putting the values in the basic integrator equation

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$$v_o = -\frac{1}{RC}\int v_i\,dt$$

$$v_o(\text{pp}) = -\frac{1}{R_1C_1}\int_0^{T/2}(-V_{sat})dt = \frac{V_{sat}}{R_1C_1}\left(\frac{T}{2}\right)$$

or,

$$T = 2R_1C_1\frac{v_o(\text{pp})}{V_{sat}} \tag{5.22}$$

Putting the value of v_o (pp) from Eq. (5.21), we get

$$T = \frac{4R_1C_1R_2}{R_3}$$

Hence the frequency of oscillation f_o is,

$$f_o = \frac{1}{T} = \frac{R_3}{4R_1C_1R_2} \tag{5.23}$$

Therefore, the triangular wave oscillates between + 7V and – 7V.

Sawtooth Wave Generator

The difference between the triangular and sawtooth waveforms is that the rise time of a triangular wave is always equal to its fall time. That is, the same amount of time is taken by the triangular wave to swing from $-V_{ramp}$ to $+V_{ramp}$ as from $+V_{ramp}$ to $-V_{ramp}$. On the other hand, the sawtooth waveform has unequal rise and fall times. That is, it may rise many times faster than it falls negatively, or vice versa. The triangular wave generator is as shown in Fig 5.14.

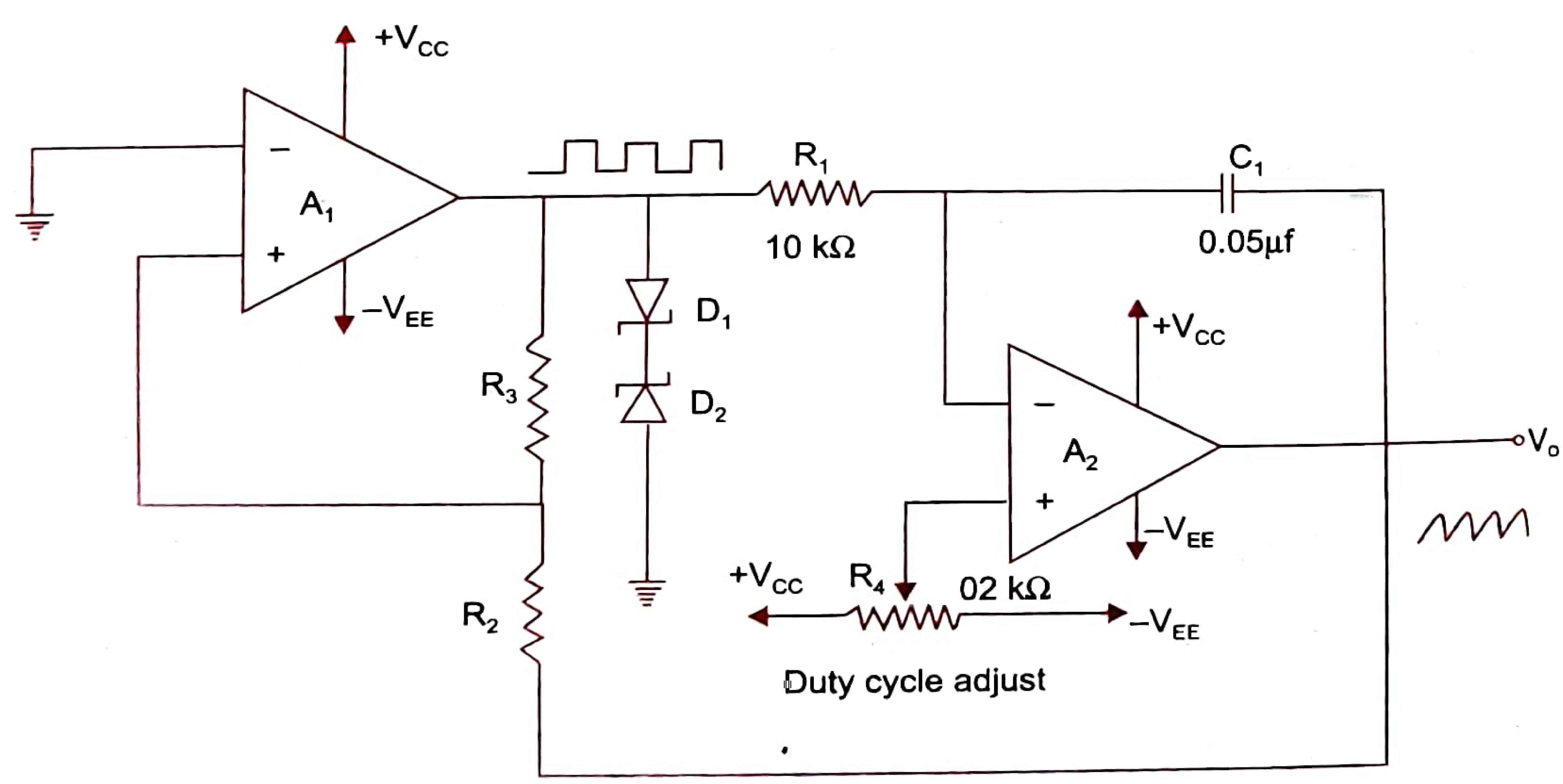

Fig. 5.14 Sawtooth Wave Generator

The triangular wave generator can be converted into a sawtooth wave generator by applying a variable dc voltage into the noninverting terminal of the integrator A_2. This can be accompalished by using the potentiometer and connecting it to the $+V_{CC}$ and $+V_{EE}$ as shown in Fig 5.14.

Depending on the R_4 setting, a certain dc level is inserted in the output of A_2. Now, suppose that the output of A_1 is a square wave and the potentiometer R_4 is adjusted for a certain dc level. This means that the output of A_2 will be a triangular wave, rising on some dc level that is a function of the R_4 setting. The duty cycle of the square wave will be determined by the polarity and amplitude of this dc level. A duty cycle less than 50% will then cause the output of A_2 to be a sawtooth. With the wiper at the center of R_4, the output of A_2 is a triangular wave. For any other position of R_4 wiper, the output is a sawtooth waveform. Specifically as the R_4 wiper is moved toward $-V_{EE}$, the rise time of the sawtooth wave becomes longer than the fall time as shown in Fig 5.15.

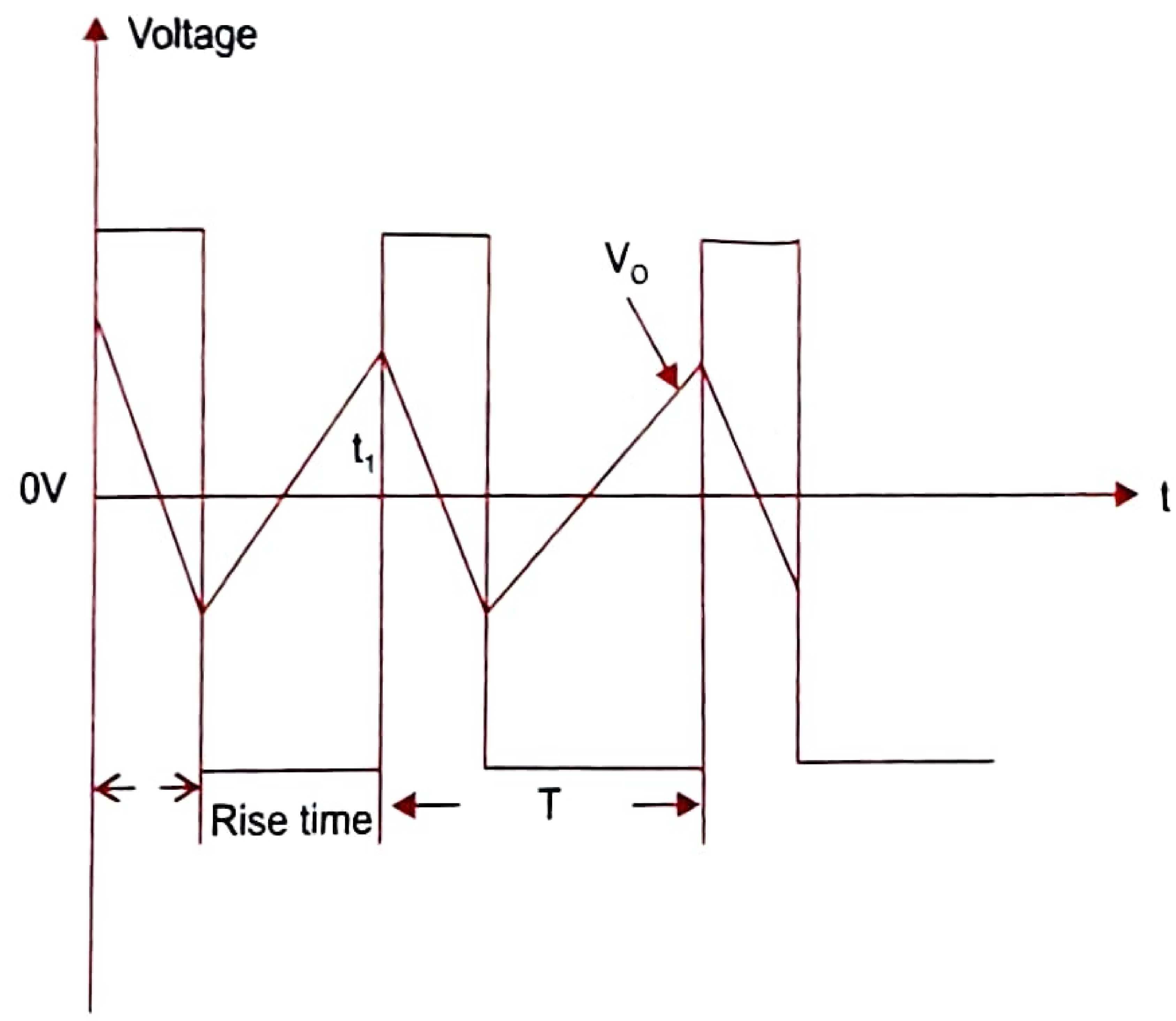

Fig. 5.15 Sawtooth waveform

On the otherhand, as the wiper is moved towards $+V_{CC}$, the fall time becomes longer than the rise time. Also, the frequency of the Sawtooth decreases as R_4 is adjusted towards $+V_{CC}$ or $-V_{EE}$

However, the amplitude of the sawtooth wave is independent of the R_4 setting.

Example 5.4

Design a sawtooth wave generator for 10V peak and frequency of 200 Hz. Assume V_i = 2V and V_{ref} = 10V.

Solution

The ramp signal rises at a rate of 2V/ms. Therefore, choose a time constant of R_1C to produce a time period of 1.0 ms.

Let $R_1 = 10\ \text{k}\Omega$ and $C = 0.1\ \mu\text{F}$. We know that $f = \left(\frac{1}{R_1C}\right)\frac{V_i}{V_{ref}}$

Therefore
$$f = \frac{1}{(10\times10^3)(0.1\times10^{-6})}\left(\frac{2}{10}\right) = 200\ \text{Hz}$$

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5.7 BASIC PRINCIPLE OF SINE WAVE OSCILLATORS

The basic structure of sine wave oscillators based on the use of feedback in amplifiers is shown in Fig. 5.16. It consists of an amplifier with gain A and a frequency selective feedback network (having inductor or capacitive components) with the transfer ratio β. It may be noted that the loop is incomplete as the terminal 2 is not connected to terminal 1. To understand the operation of the circuit, consider the situation where an input signal v_i is applied at the input terminal 1 of the amplifier, so that the output $v_o = A\, v_i$. The feedback signal v_f at terminal 2, therefore is $v_f = A\beta\, v_1$. The quantity Aβ, therefore, represents the loop gain of the system. If the values of A and β are adjusted so that Aβ = 1, the feedback signal v_f will be identically equal to the externally applied signal v_i. If the terminal 2 is now connected to terminal 1 and the external signal v_i is removed, the circuit will continue to provide output as the amplifier can not distinguish whether v_i is coming from external source or from the feedback circuit. Thus, output signal can be continuously obtained without any input signal if we can satisfy the condition on the loop gain, that is,

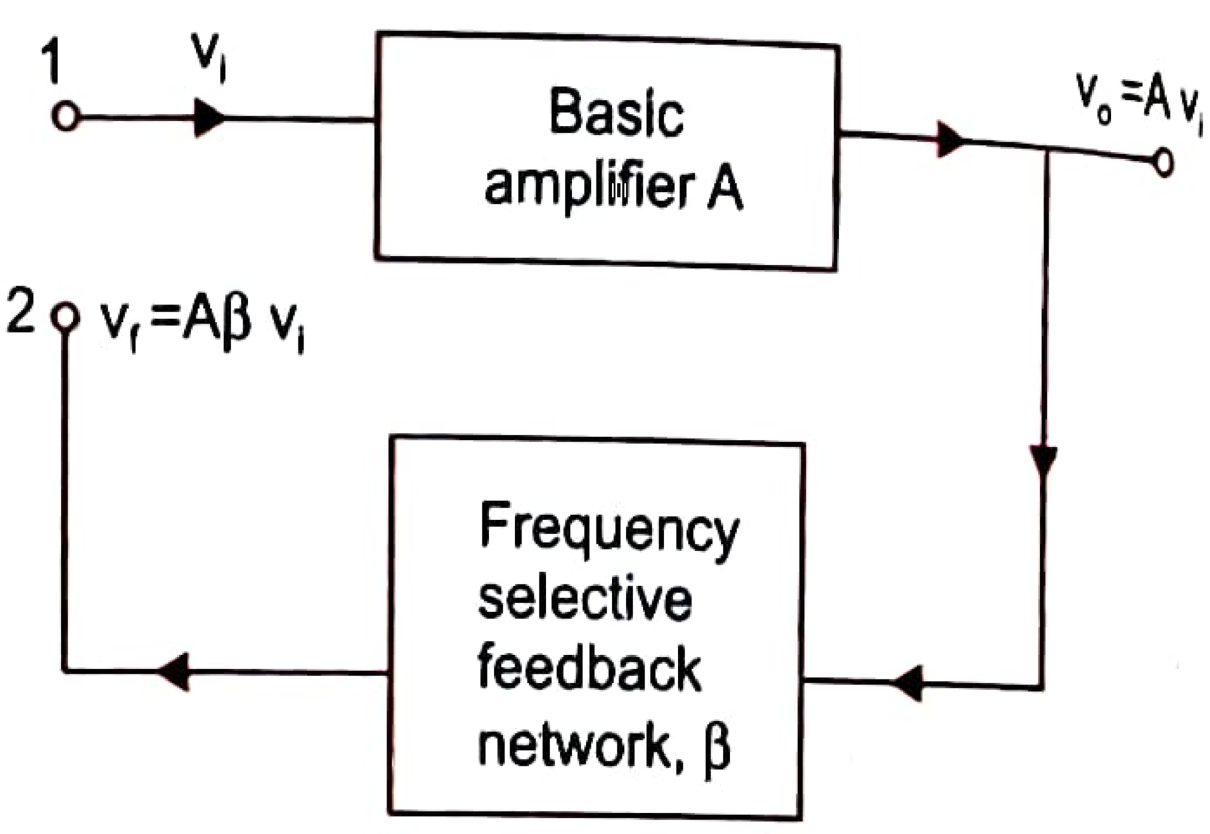


Fig. 5.16 Basic structure of a feedback oscillator

$$A\beta = 1 \tag{5.24}$$

This is called ***Barkhausen criterion*** for oscillations. The condition Aβ = 1 can be satisfied only at one specific frequency, f_o for the given component values. The circuit thus provides output at frequency, f_o where the circuit components meets the condition given by Eq. (5.24). We may rewrite Eq. (5.24) as

$$A(j\omega_o)\ \beta(j\omega_o) = 1\ \angle 0° \tag{5.25}$$

There are infact two conditions in Eq. (5.25), one on phase and other on the magnitude of the loop gain which needs to be simultaneously satisfied to achieve oscillations. Thus, according to Eq. (5.25) the total phase shift of the loop gain should be zero or multiples of 2π and the magnitude of the loop gain, Aβ should be equal to unity. That is

$$|A\beta| = 1 \tag{5.26}$$

$$\angle A\beta = 0° \text{ or multiples of } 2\pi \tag{5.27}$$

The condition $|A\beta| = 1$ is usually difficult to maintain in the circuit as the values of A and β vary due to temperature variations, aging of components, change of supply voltage etc. If $|A\beta|$ becomes less than unity, the feedback signal v_f goes on reducing in each feedback cycle and the oscillations will die down eventually. In order to ensure that the circuit sustains oscillations inspite of variations, the circuit is designed so that $|A\beta|$ is slightly greater than unity. Now, the output amplitude will go on increasing with every feedback cycle. The signal, however, can not go on increasing and gets limited due to the non-linearity of the device, that is as the transistor enters into saturation. Thus it is the non-linearity of the transistor because of which the sustained oscillations can be achieved. The value of Aβ is usually kept greater by about 1 to 5% to ensure that $|A\beta|$ does not fall below unity. In explaining the principle of oscillation in Fig. 5.14, we had assumed that we first connect a signal source to start the oscillations and later remove it. In a practical oscillator, however, it is not done so. The output waveform is obtained as soon as power is turned on. Actually, there is noise signal always present at the input (i.e. base) of the transistor due to temperature (called Johnson's noise) or variation in the carrier concentration (Schottky noise). The noise signal at the frequency at which the circuit satisfies the condition $|A\beta| = 1$ is picked up and amplified. Since $|A\beta| > 1$ in the circuit, the output signal goes on increasing until it is limited by the onset of non-linearity of the transistor (as transistor enters into saturation) as shown in Fig. 5.15.

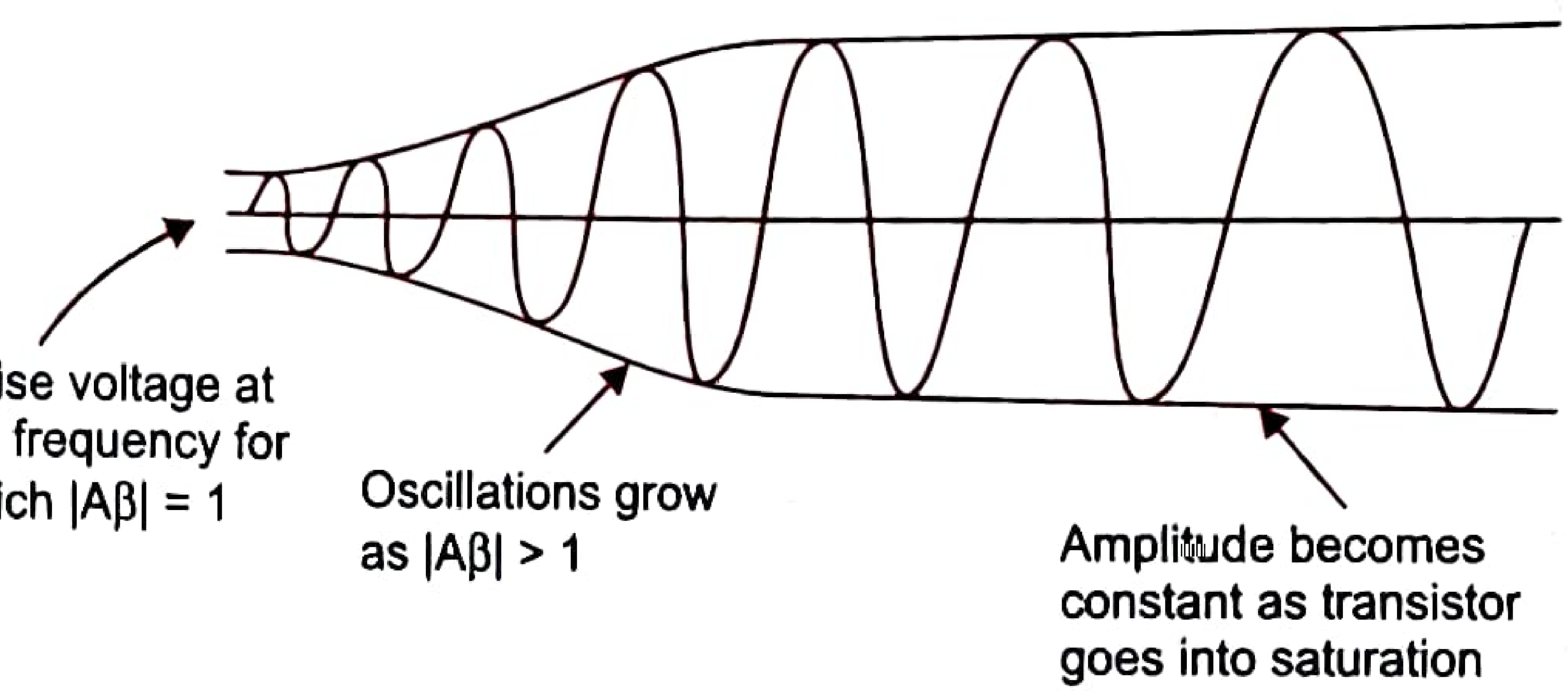


Fig. 5.17 Showing constant output amplitude as transistor goes into saturation ($|A\beta| > 1$)

There are different types of sine-wave oscillators available according to the range of frequency. The RC-phase shift oscillators can provide frequencies varying from a few hertz to several hundred kHz. LC oscillators are suitable for high frequencies up to hundreds of MHz. Here we will discuss only two types of audio frequency RC phase shift oscillators.

RC-Phase Shift Oscillator

The circuit of an RC-phase shift oscillator is shown in Fig. 5.16 (a). The op-amp is used in the inverting mode and therefore provides 180° phase shift. The additional phase of 180° is provided by the RC feedback network to obtain a total phase shift of 360°. The feedback network consists of three identical RC stages. Each of the RC stage provides a 60° phase shift so that the total phase shift due to feedback network is 180°. It is not necessary that all the three RC sections are identical so long the total phase shift is 180°. However, if we use non-identical stages, it is possible that the total phase shift is 180° for more than one frequency. This phenomenon can lead to undesirable inter-modal oscillations.

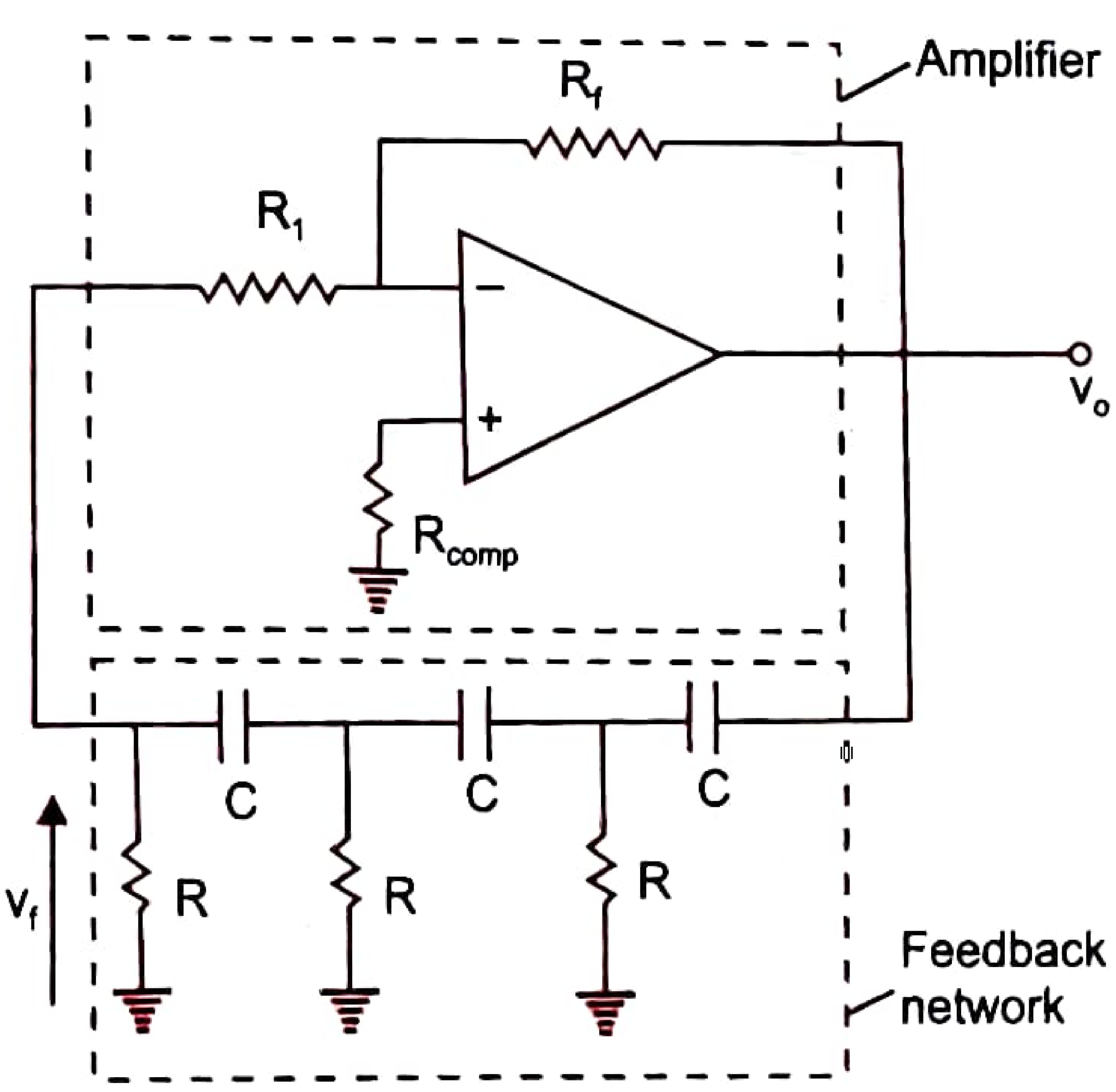


Fig. 5.18 (a) Phase shift oscillator

The feedback factor β of the RC network can be calculated by writing the KVL equations from Fig. 5.16 (b).

$$I_1\left(R+\frac{1}{sC}\right)-I_2R = V_o \quad (5.28)$$

$$-I_1R+I_2\left(2R+\frac{1}{sC}\right)-I_3R = 0 \quad (5.29)$$

$$0-I_2R+I_3\left(2R+\frac{1}{sC}\right) = 6 \quad (5.30)$$

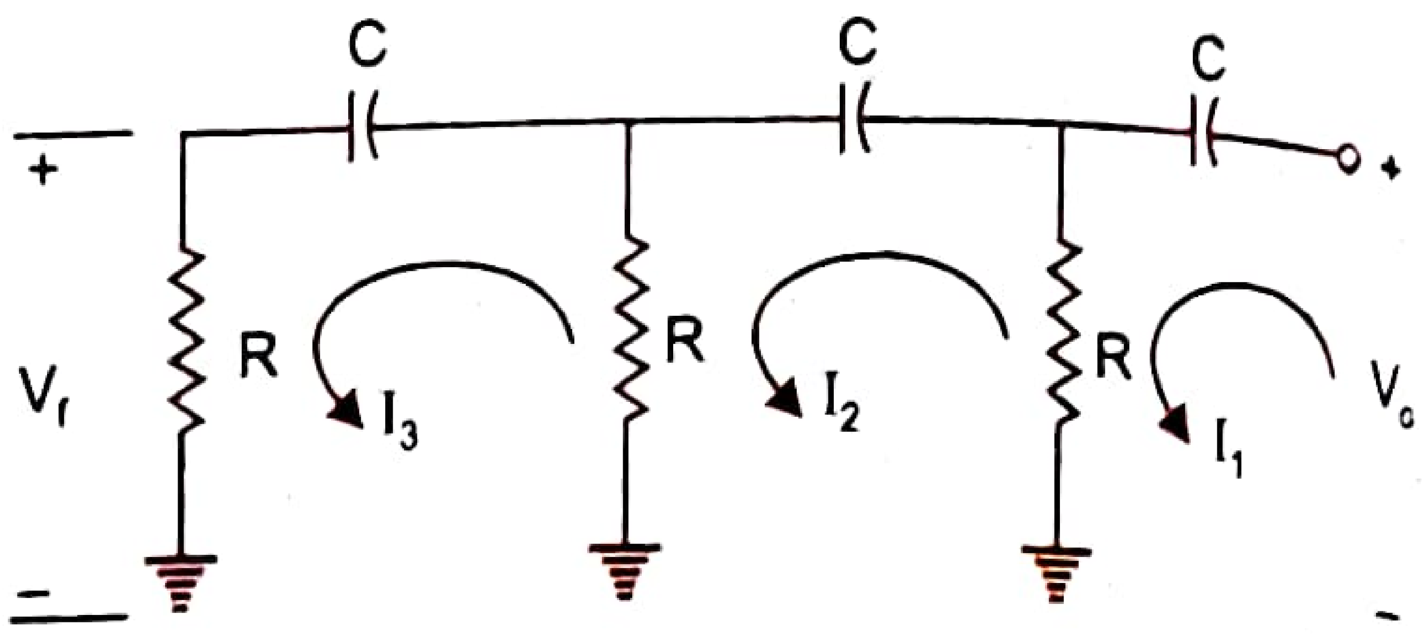


Fig. 5.18 (b) Calculating β from the phase shift network

and $$V_f = I_3R \quad (5.31)$$

Solving Eqs. (5.28), (5.29) and (5.30) for I_3, we get

$$I_3 = \frac{V_oR^2s^3C^3}{1+5sRC+6s^2C^2R^2+s^3C^3R^3} \quad (5.32)$$

and $$V_f = I_3R = \frac{V_oR^3s^3C^3}{1+5sRC+6s^2C^2R^2+s^3C^3R^3} \quad (5.33)$$

$$= \frac{1}{1+\dfrac{6}{sRC}+\dfrac{5}{s^2C^2R^2}+\dfrac{1}{s^3C^3R^3}} \quad (5.34)$$

Replacing $s = j\omega$, $s^2 = -\omega^2$ and $s^3 = -j\omega^3$, we get

$$\beta = \frac{1}{1-\dfrac{6}{j\omega RC}-\dfrac{5}{\omega^2R^2C^2}+\dfrac{1}{j\omega^3R^3C^3}} \quad (5.35)$$

$$= \frac{1}{(1-5\alpha^2)+j\alpha(6-\alpha^2)} \quad (5.36)$$

where $$\alpha = \frac{1}{\omega RC} \quad (5.37)$$

For Aβ = 1, β should be real, that is the imaginary term in Eq. (5.36) must be zero. Thus

$$\alpha\,(6-\alpha^2) = 0 \quad (5.38)$$

or, $$\alpha^2 = 6$$

$$\alpha = \sqrt{6}$$

That is, $$\frac{1}{\omega RC} = \sqrt{6}$$

The frequency of oscillation, f_o, is therefore given by

$$f_o = \frac{1}{2\pi RC\sqrt{6}} \quad (5.39)$$

Putting $\alpha^2 = 6$ in Eq. (5.36), we get

$$\beta = -\frac{1}{29} \tag{5.40}$$

The negative sign indicates that the feedback network produces a phase shift of 180°.

So, $|\beta| = \frac{1}{29}$

Since $|A\beta| \geq 1$

Therefore, for sustained oscillations,

$$|A| \geq 29 \tag{5.41}$$

That is the gain of the inverting op-amp should be atleast 29, or $R_f = 29\ R_1$. The gain A_v is kept greater than 29 to ensure that variations in circuit parameters will not make $|A_v\beta| < 1$, otherwise oscillations will die out.

For low frequencies (< 1 kHz), op-amp 741 may be used, however, for high frequencies, LM 318 or LF 351 should be used.

Example 5.5

Design a phase shift oscillator of Fig. 5.17 to oscillate at 100 Hz.

Solution

Let $C = 0.1\ \mu F$. Then from Eq. (5.25)

$$R = \frac{1}{\sqrt{6}\ 2\pi(10^{-7})(100)} = 6.49\ k\Omega$$

Use $R = 6.5\ k\Omega$

To prevent loading of the amplifier by RC network, $R_1 \leq 10\ R$

Therefore, let $R_1 = 10\ R = 65\ k\Omega$

Since $R_f = 29\ R_1$

$R_f = 1885\ k\Omega$

Wien Bridge Oscillator

Another commonly used audio frequency oscillator is a Wien bridge oscillator. The circuit is shown in Fig. 5.19. It may be noted that the feedback signal in this circuit is connected to the non-inverting (+) input terminal so that the op-amp is working as a non-inverting amplifier. Therefore, the feedback network need not provide any phase shift. The circuit can be viewed as a Wien bridge with a series RC network in one arm and a parallel RC network in the adjoining arm. Resistors R_1 and R_F are connected in the remaining two arms. The condition of zero phase shift around the circuit is achieved by balancing the bridge.

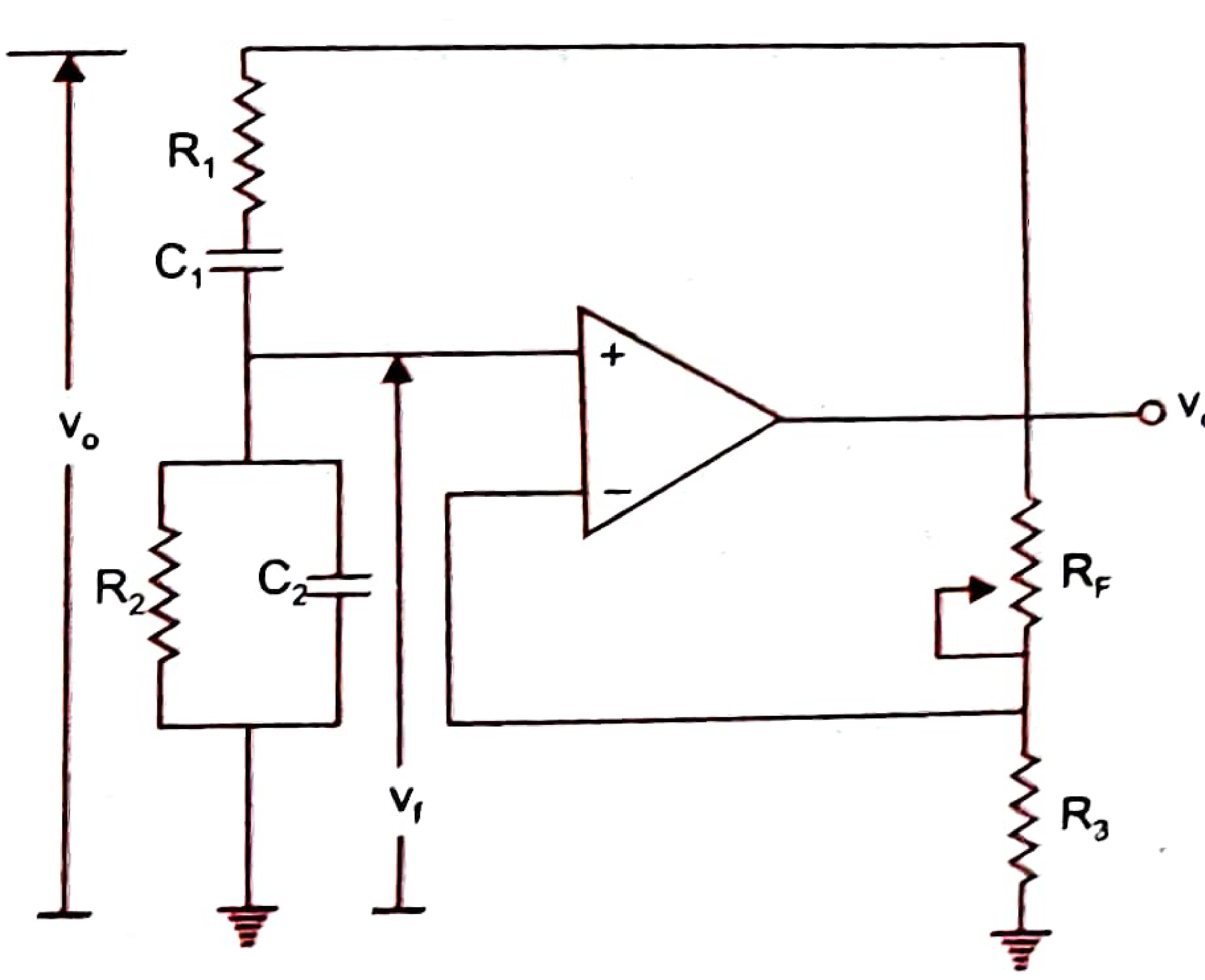


Fig. 5.19 Wien bridge oscillator

The circuit has been redrawn to show the bridge network in Fig. 5.20. The output ac signal of the op-amp amplifier is fed back to point *A* of the bridge. The feedback signal, V_f across the parallel combination R_2C_2 is applied to the non-inverting input terminal of the op-amp. The gain of the op-amp amplifier is

$$A = 1 + \frac{R_F}{R_3} \quad (5.42)$$

and feedback factor, β from Fig. 5.18 is

$$\beta = \frac{V_f}{V_o} = \frac{Z_2}{Z_1 + Z_2} \quad (5.43)$$

where $Z_1 = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$ (5.44)

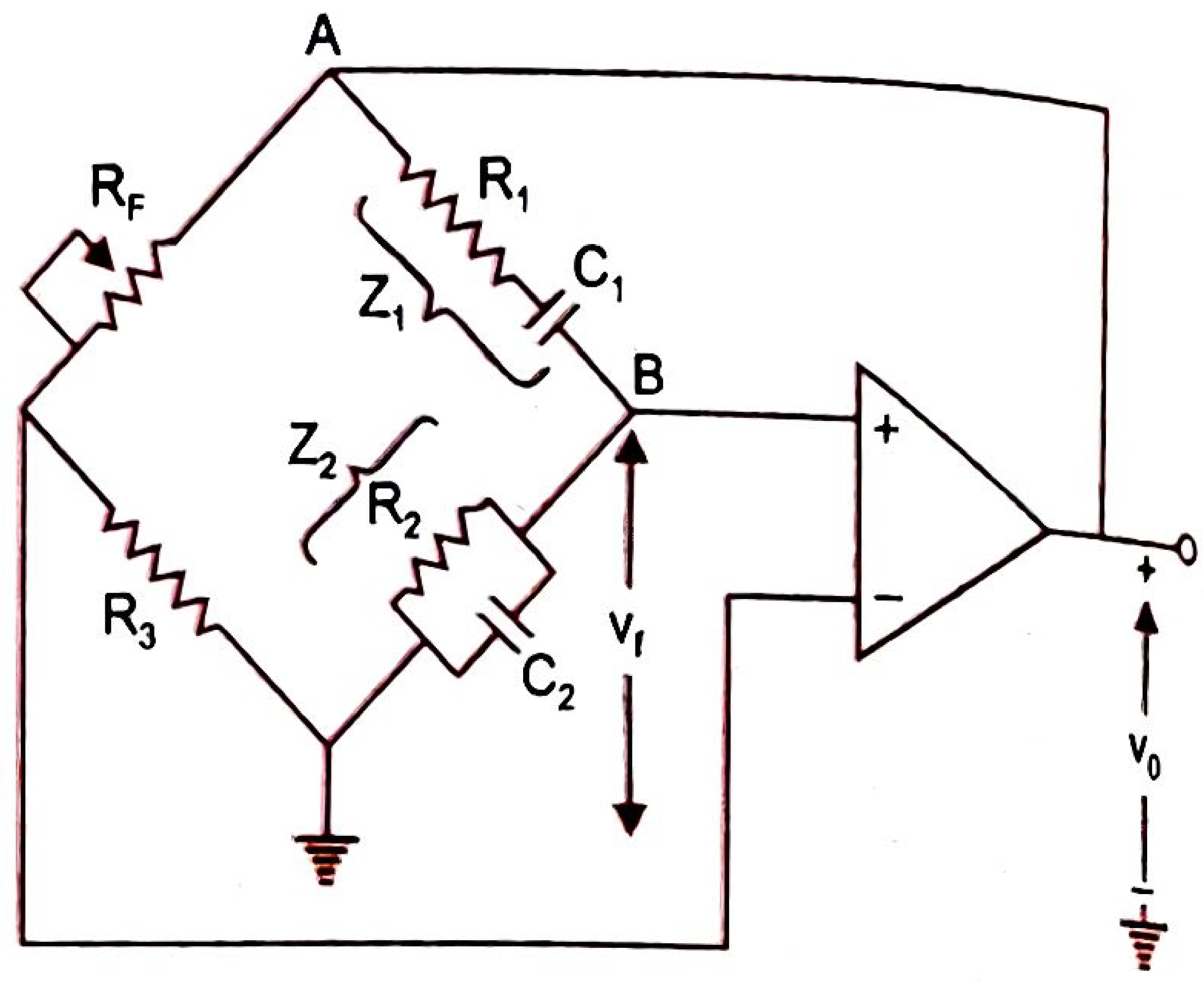


Fig. 5.20 Wien bridge oscillator showing the bridge network

$$Z_2 = \frac{R_2}{1 + sR_2C_2} \quad (5.45)$$

Putting the values of Z_1 and Z_2 in Eq.(5.43), we get

$$\beta = \frac{R_2/(1 + R_2C_2s)}{\frac{1 + sR_1C_1}{sC_1} + \frac{R_2}{1 + sR_2C_2}} \quad (5.46)$$

$$= \frac{sR_2C_1}{1 + s(R_1C_1 + R_2C_2 + R_2C_1) + s^2R_1R_2C_1C_2} \quad (5.47)$$

Putting $s = j\omega$,

$$\beta = \frac{j\omega R_2C_1}{1 + j\omega(R_1C_1 + R_2C_2 + R_2C_1) - \omega^2R_1R_2C_1C_2} \quad (5.48)$$

In order β to be a real quantity

$$1 - \omega^2R_1R_2C_1C_2 = 0$$

Thus, the frequency of oscillation,

$$f_o = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} \quad (5.49)$$

and

$$\beta = \frac{R_2C_1}{R_1C_1 + R_2C_2 + R_2C_1} \quad (5.50)$$

For $R_1 = R_2 = R$ and $C_1 = C_2 = C$,

$$f_o = \frac{1}{2\pi RC} \tag{5.51}$$

and

$$\beta = \frac{1}{3} \tag{5.52}$$

Since $|A\beta| \geq 1$ for sustained oscillations,

$$|A| \geq 3$$

Since

$$A = 1 + \frac{R_F}{R_3}$$

$$3 = 1 + \frac{R_F}{R_3}$$

or,

$$R_F = 2R_3 \quad (5.53)$$

If the gain $|A| > 3$, sometimes oscillations keep growing and it may clip the output sinewave. This problem is eliminated by a practical Wien bridge oscillator with adaptive negative feedback as shown in Fig. 5.21. In this circuit, resistor R_4 is initially adjusted to give a gain so that oscillations start. The output signal grows in amplitude until the voltage across R_3 approaches the cut-in voltage of the diode. As the diodes begin to turn-on (one for the positive half cycle and the other for the negative half cycle), the effective feedback resistance R_F decreases because the diode is in parallel with the resistance R_3. This will reduce the gain of the amplifier which in turn lowers the output amplitude. Hence sustained oscillations can be obtained. Further, if the output signal falls, the diodes would begin to turn-off thereby increasing R_F which in turn increases gain.

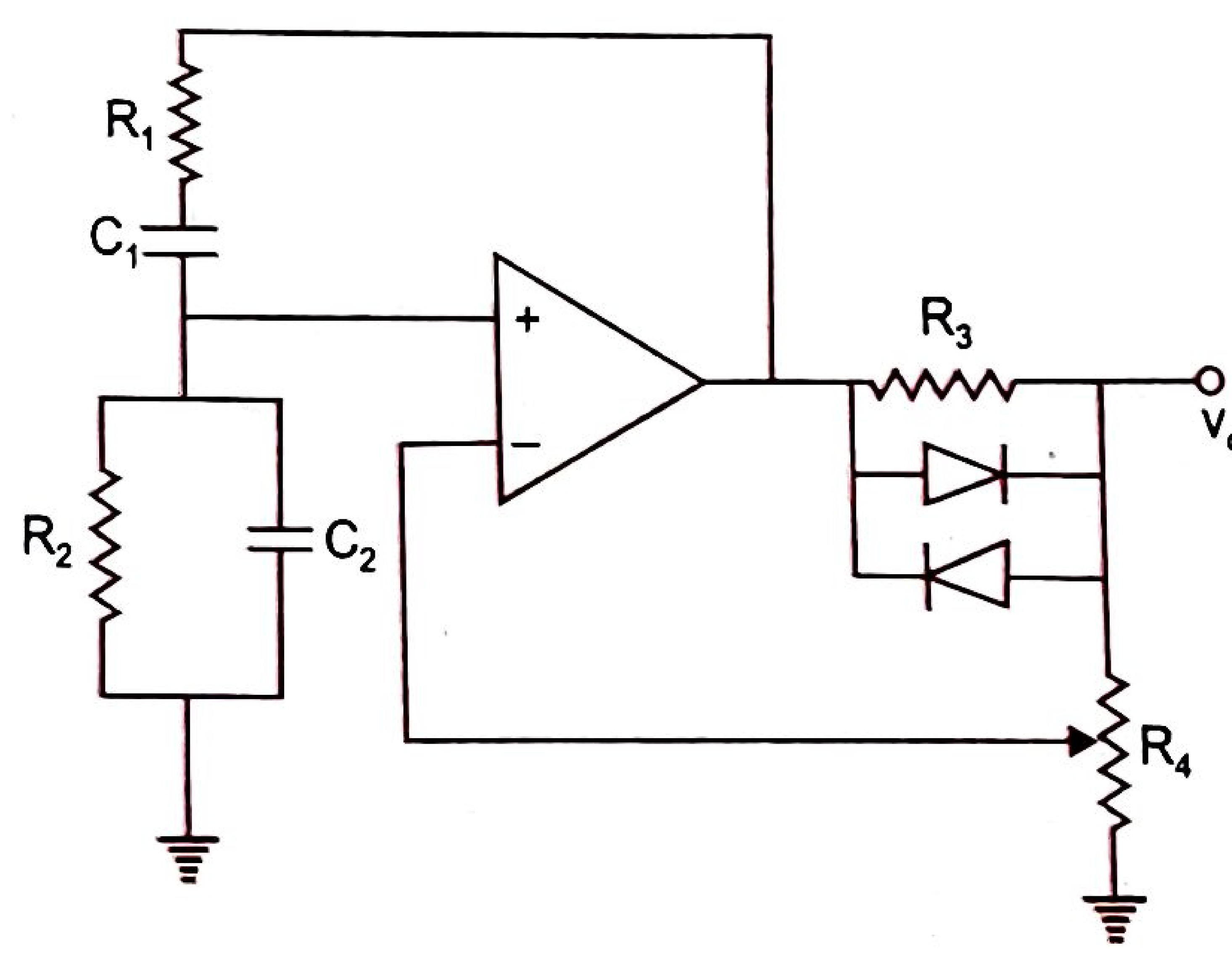


Fig. 5.21 Practical Wien bridge oscillator with adaptive negative feedback

The two op-amp *RC* oscillator circuits studied are suitable for the frequency range of 10 Hz to 100 kHz (maximum 1 MHz). The size of *R* and *C* components becomes very large for generating low frequencies. Thus, the low frequency limit is dictated by the size of passive components required. The upper frequency limit is governed by the frequency response and slew rate limit of the op-amp used. For generating high frequencies in the *RF* range, the oscillator circuits are usually designed using BJT and LC tuned circuits or crystal oscillators.

LC Oscillators

For generating high frequency sine waves in the *RF* range, *LC* tuned circuits are used. The most commonly used *LC* oscillators are Colpitts oscillator and Hartley oscillator which are discussed here.

7

ACTIVE FILTERS

7.1 INTRODUCTION

Electric filters are used in circuits which require the separation of signals according to their frequencies. Filters are widely used in communication and signal processing and in one form or another in almost all sophisticated electronic instruments. Such filters can be built from, (i) passive RLC components, (ii) crystals or (iii) resistors, capacitors and op-amps (active filters). In this chapter, we are discussing (i) RC active filters and (ii) switched capacitor filters. Further, active filters in its low-pass, high-pass, band-pass, band elimination configura-tion and state variable filter have been discussed.

7.2 RC ACTIVE FILTERS

A frequency selective electric circuit that passes electric signals of specified band of frequencies and attenuates the signals of frequencies outside the band is called an electric filter. Filters may be analog or digital. Our point of discussion, in this chapter, will be analog filters.

The simplest way to make a filter is by using passive components (resistors, capacitors, inductors). This works well for high frequencies, that is, radio frequencies. However, at audio frequencies, inductors become problematic, as the inductors become large, heavy and expensive. For low frequency application, more number of turns of wire must be used which in turn adds to the series resistance degrading inductor's performance, i.e. low Q, resulting in high power dissipation.

The active filters overcome the aforementioned problems of the passive filters. They use op-amp as the active element, and resistors and capacitors as the passive elements. The active filters, by enclosing a capacitor in the feedback loop, avoid using inductors. In this way, inductorless active RC filters can be obtained. Op-amps filters have the advantage that they can provide gain. Thus, the input signal is not attenuated as in the case of passive filters. Also, as op-amp is used in non-inverting configuration, it offers high input impedance and low output impedance. This will improve the load drive capacity and the load is isolated from the frequency determining network. Because of the high input impedance of the op-amp, large value resistors can be used, thereby reducing the value (size and cost) of the capacitors required in the design.

The active filters have their limitation too. High frequency response is limited by the gain-bandwidth (GBW) product and slew rate of the op-amp. Moreover, the high frequency active filters are more expensive than the passive filters. The passive filter in high frequency range is a more economic choice for applications.

The most commonly used filters are:

Low Pass Filter	(LPF)
High Pass Filter	(HPF)
Band Pass Filter	(BPF)
Band Reject Filter	(also called Band Stop Filter) (BSF)

The frequency response of these filters is shown in Fig. 7.1, where dashed curve indicates the ideal response and solid curve shows the practical filter response. It is not possible to achieve ideal characteristics. However, with special design techniques it is possible to closely approximate the ideal response.

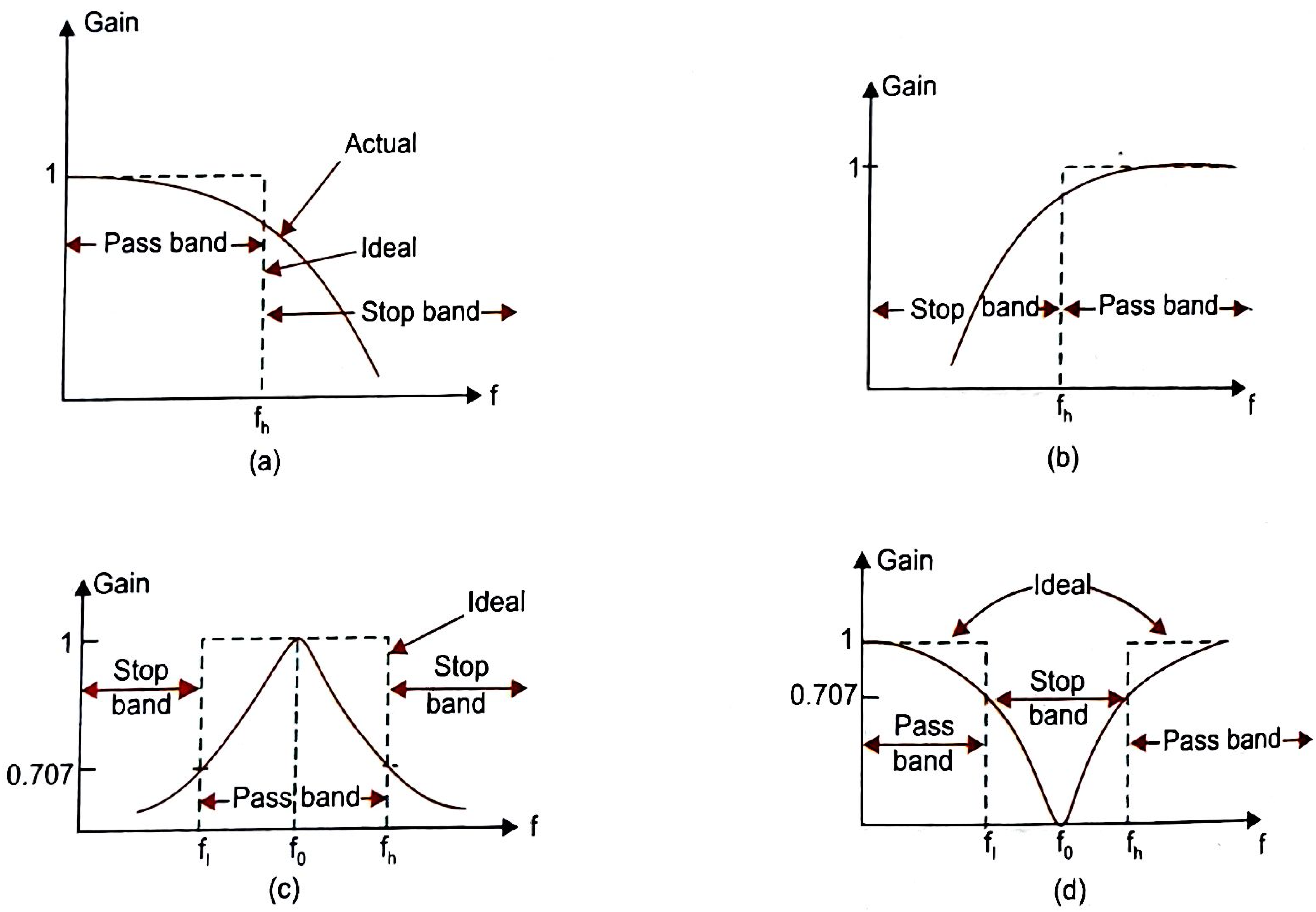


Fig. 7.1 Frequency response of filters, (a) Low-pass, (b) High-pass, (c) Band-pass, (d) Band-reject

Active filters are typically specified by the voltage transfer function,

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

Under steady state conditions (i.e., $s = j\omega$)

$$H(j\omega) = |H(j\omega)|\, e^{j\phi(\omega)} \tag{7.1}$$

where $|H(j\omega)|$ is the magnitude or the gain function and $\phi(\omega)$ is the phase function. Usually the magnitude response is given in dB as

$$20 \log |H(j\omega)| \tag{7.2}$$

and the phase response is given in degrees as

$$-\phi(\omega) \times 57.296 \text{ degrees} \tag{7.3}$$

Sometimes, active filters are specified by a loss function $V_i(s)/V_o(s)$. The use of loss function is a carry over from passive filter design.

7.2.1 First Order Low Pass Filter

Active filters may be of different orders and types. A first order filter consists of a single RC network connected to the (+) input terminal of a non-inverting op-amp amplifier and is shown in Fig. 7.2 (a). Resistors R_i and R_F determine the gain of the filter in the pass band.

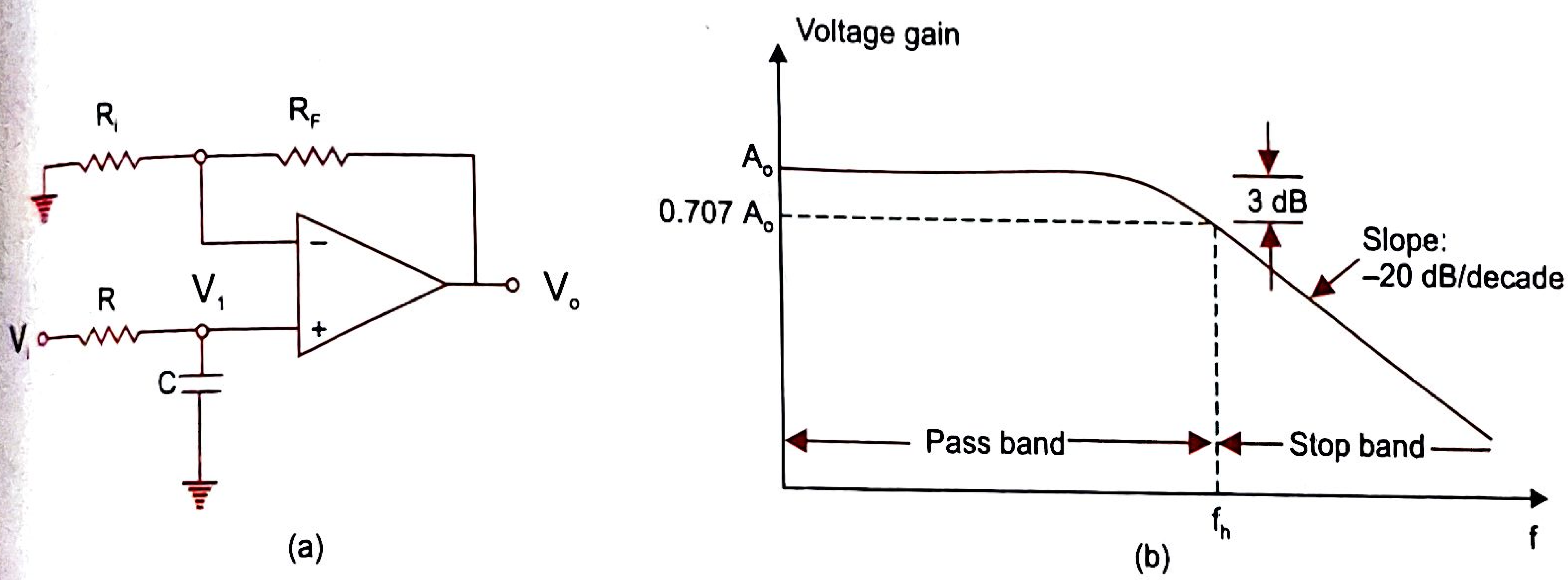


Fig. 7.2 (a) First order low-pass filter, (b) Frequency response

The voltage V_1 across the capacitor C in the s-domain is

$$V_1(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i(s)$$

So,

$$\frac{V_1(s)}{V_i(s)} = \frac{1}{RCs+1} \tag{7.4}$$

where $V(s)$ is the Laplace transform of v in time domain.

The closed loop gain A_o of the op-amp is,

$$A_o = \frac{V_o(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_i}\right) \tag{7.5}$$

So, the overall transfer function from Eqs. (7.4) and (7.5) is

$$H_{LP}(s) = \frac{V_o(s)}{V_i(s)} = \frac{V_o(s)}{V_1(s)} \cdot \frac{V_1(s)}{V_i(s)} = \frac{A_o}{RCs+1} \tag{7.6}$$

Let

$$\omega_h = \frac{1}{RC} \tag{7.7}$$

Therefore, $$H_{LP}(s) = \frac{V_o(s)}{V_i(s)} = \frac{A_o}{\dfrac{s}{\omega_h}+1} = \frac{A_o\,\omega_h}{s+\omega_h} \qquad (7.8)$$

This is the standard form of the transfer function of a first order low pass system. To determine the frequency response, put $s = j\omega$ in Eq. (7.8). Therefore, we get

$$H_{LP}(j\omega) = \frac{A_o}{1+j\omega RC} = \frac{A_o}{1+j(f/f_h)} \qquad (7.9)$$

where $$f_h = \frac{1}{2\pi RC} \text{ and } f = \frac{\omega}{2\pi}$$

At very low frequency, i.e. $f << f_h$

$$|H_{LP}(j\omega)| \simeq A_o \qquad (7.10)$$

At $$f = f_h,$$

$$|H_{LP}(j\omega)| = \frac{A_o}{\sqrt{2}} = 0.707\,A_o \qquad (7.11)$$

At very high frequency i.e. $f >> f_h$

$$|H_{LP}(j\omega)| << A_o \simeq 0 \qquad (7.12)$$

The frequency response of the first order low pass filter is shown in Fig. 7.2 (b). It has the maximum gain, A_o at $f = 0$ Hz. At f_h the gain falls to 0.707 time (i.e. –3 dB down) the maximum gain (A_o). The frequency range from 0 to f_h is called the pass band. For $f > f_h$ the gain decreases at a constant rate of –20 dB/decade. That is, when the frequency is increased ten times (one decade), the voltage gain is divided by ten or in terms of dBs, the gain decreases by 20 dB (= 20 log 10). Hence, gain rolls off at the rate of 20 dB/decade or 6 dB/octave after frequency, f_h. The frequency range $f > f_h$ is called the stop band. Obviously, the low pass filter characteristics obtained is not an ideal one as the rate of decay is small for the first order filter.

7.2.2 Second Order generalised Active Filter (Sallen-key filter)

An improved filter response can be obtained by using a second order active filter. A second order filter consists of two RC pairs and has a roll-off rate of – 40 dB/decade. A general second order filter (Sallen-Key filter) is shown in Fig. 7.3. The results derived here can be used for analysing low pass and high pass filters.

The op-amp is connected as non-inverting amplifier and hence,

$$v_o = \left(1+\frac{R_F}{R_i}\right)v_B = A_o\,v_B \qquad (7.13)$$

where $$A_o = 1+\frac{R_F}{R_i} \qquad (7.14)$$

and v_B is the voltage at node B.

Kirchhoff's current law (KCL) at node A gives

$$v_i Y_1 = v_A(Y_1 + Y_2 + Y_3) - v_o Y_3 - v_B Y_2$$

$$= v_A(Y_1 + Y_2 + Y_3) - v_o Y_3 - \frac{v_o Y_2}{A_o} \quad (7.15)$$

where v_A is the voltage at node A.

KCL at node B gives,

$$v_A Y_2 = v_B(Y_2 + Y_4) = \frac{v_o(Y_2 + Y_4)}{A_o}$$

$$v_A = \frac{v_o(Y_2 + Y_4)}{A_o Y_2} \quad (7.16)$$

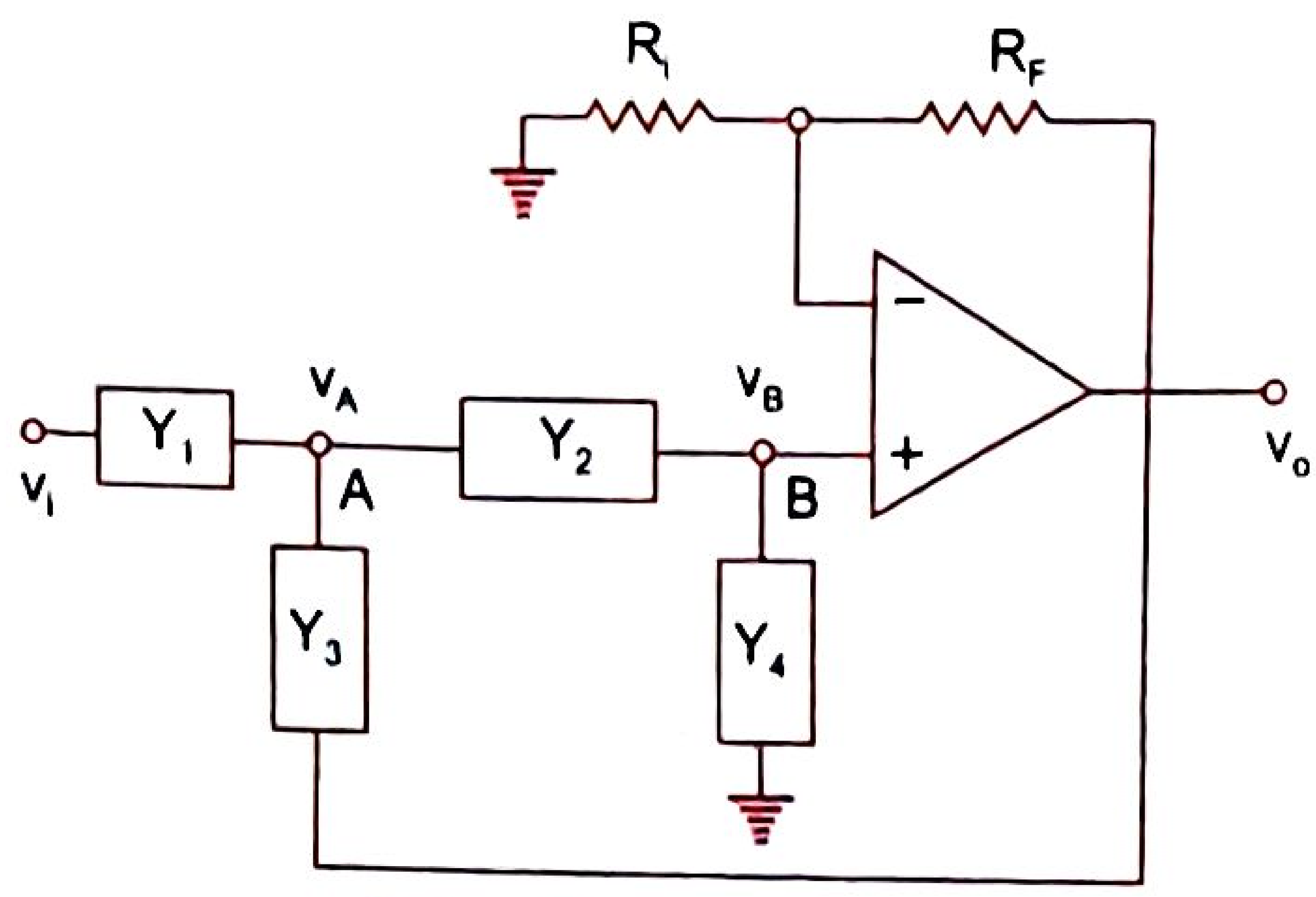


Fig. 7.3 Sallen-Key filter (General second order filter)

Substituting Eq. (7.16) in Eq. (7.15) and after simplification, we get the voltage gain as

$$\frac{v_o}{v_i} = \frac{A_o Y_1 Y_2}{Y_1 Y_2 + Y_4(Y_1 + Y_2 + Y_3) + Y_2 Y_3 (1 - A_o)} \quad (7.17)$$

To make a low pass filter, choose, $Y_1 = Y_2 = 1/R$ and $Y_3 = Y_4 = sC$ as shown in Fig. 7.4. For simplicity, equal components have been used.

From Eq. (7.17), we get the transfer function $H(s)$ of a low pass filter as,

$$H(s) = \frac{A_o}{s^2 C^2 R^2 + sCR(3 - A_o) + 1} \quad (7.18)$$

This is to note that from Eq. (7.18), $H(0) = A_o$ for $s = 0$ and $H(\infty) = 0$ for $s = \infty$ and obviously the configuration is for low pass active filter. It may be noted that for minimum dc offset $R_i R_F/(R_F + R_i) = R + R = 2R$ should be satisfied.

Second order physical systems have been studied extensively since long back and their step response, damping coefficient and its cause and effect relationship are known. We shall exploit those ideas in case of second order RC active filter. The transfer function of low pass second order system (electrical, mechanical, hydraulic or chemical) can be written as,

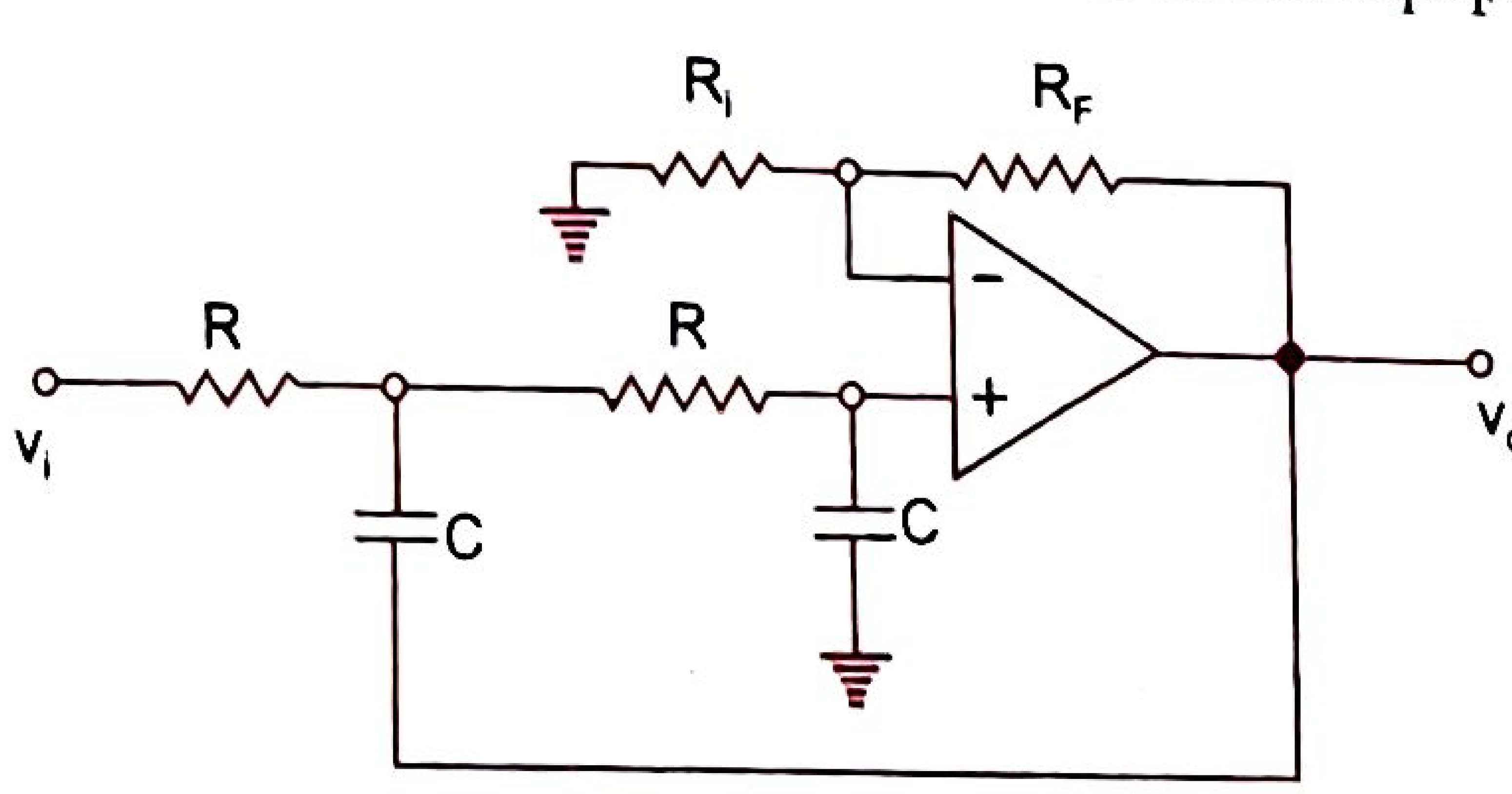


Fig. 7.4 Second order low-pass filter

$$H_{LP}(s) = \frac{A_o \, \omega_h^2}{s^2 + \alpha \omega_h s + \omega_h^2} \quad (7.19)$$

where A_o = the gain

ω_h = upper cut-off frequency in radians/second

α = damping coefficient

Comparing Eq. (7.18) and Eq. (7.19), we get,

$$\omega_h = \frac{1}{RC} \tag{7.20}$$

$$\alpha = (3 - A_o) \tag{7.21}$$

That is, the value of the damping coefficient α for low pass active *RC* filter can be determined by the value of A_o chosen.

Putting $s = j\omega$ in Eq. (7.19), we get

$$H_{LP}(j\omega) = \frac{A_o}{(j\omega/\omega_h)^2 + j\alpha(\omega/\omega_h) + 1} \tag{7.22}$$

the normalized expression for low pass filter is

$$H_{LP}(j\omega) = \frac{A_o}{s_n^2 + \alpha s_n + 1} \tag{7.23}$$

where normalized frequency $s_n = j\left(\frac{\omega}{\omega_h}\right)$

The expression of magnitude in dB of the transfer function is,

$$20 \log|H(j\omega)| = 20 \log\left|\frac{A_o}{1 + j\alpha(\omega/\omega_h) + (j\omega/\omega_h)^2}\right|$$

$$= 20 \log \frac{A_o}{\sqrt{\left(1 - \frac{\omega^2}{\omega_h^2}\right)^2 + \left(\alpha \frac{\omega}{\omega_h}\right)^2}} \tag{7.24}$$

The frequency response for different values of α is shown in Fig. 7.5. It may be seen that for a heavily damped filter (α > 1.7), the response is stable. However, the roll-off begins very

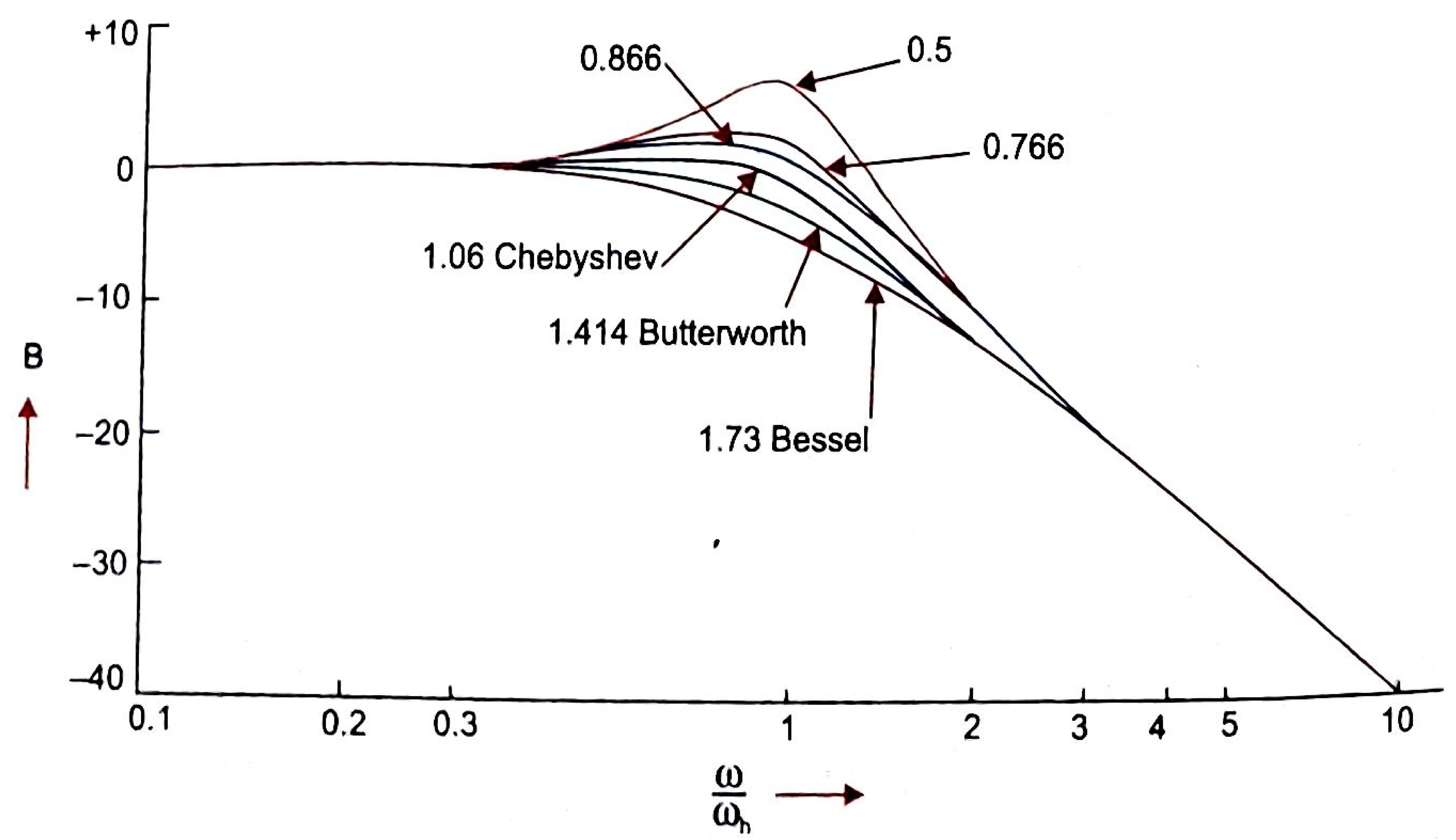


Fig. 7.5 Second order low-pass active filter response for different damping (unity gain A_o = 1)

early to the pass band. As α is reduced, the response exhibits overshoot and ripple begins to appear at the early stage of pass band. If α is reduced too much, the filter may become oscillatory. The flattest pass band occurs for damping coefficient of 1.414. This is called a ***Butterworth filter***. Audio filters are usually Butterworth. The ***Chebyshev filters*** are more lightly damped, that is, the damping coefficient α is 1.06. However, this increases overshoot and ringing occurs deteriorating the pulse response. The advantage, however, is a faster initial roll-off compared to Butterworth. A ***Bessel filter*** is heavily damped and has a damping coefficient of 1.73. This gives better pulse response, however, causes attenuation in the upper end of the pass band.

We shall discuss only Butterworth filter in this text as it has maximally flat response with damping coefficient α = 1.414. From Eq. 7.24, with α = 1.414, we get

$$20\log|H_{LP}(j\omega)| = 20\log\left|\frac{V_o}{V_i}\right| = 20\log\frac{A_o}{\sqrt{1+\left(\frac{\omega}{\omega_h}\right)^4}} \quad (7.25)$$

Hence for n-th order generalized low-pass Butterworth filter, the normalized transfer function for maximally flat filter can be written as

$$\left|\frac{H_{LP}(j\omega)}{A_o}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_h}\right)^{2n}}} \quad (7.26)$$

7.2.3 Higher Order Low Pass Filter

A second order filter can provide –40 dB/decade roll-off rate in the stop band. To match with ideal characteristics, the roll-off rate should be increased by increasing the order of the filter. Each increase in order will produce –20 dB/decade additional increase in roll-off rate, as shown in Fig. 7.6. For n-th order filter the roll-off rate will be $-n \times 20$ dB/decade.

Higher order filters can be built by cascading a proper number of first and second order filters. The transfer function will be of the type,

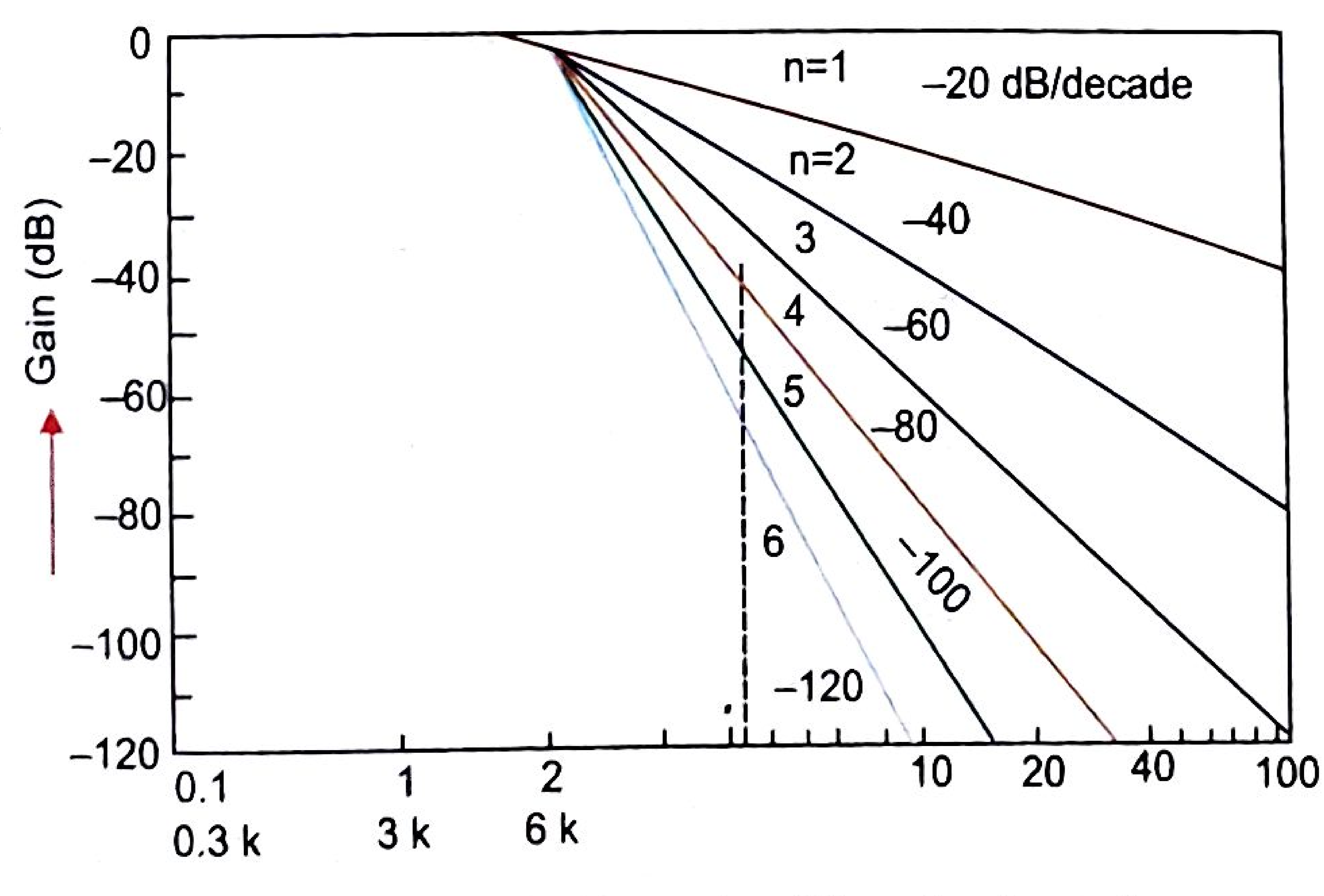


Fig. 7.6 Roll-off rate for different values of n

$$H(s) = \underset{\text{second order section}}{\frac{A_{o1}}{s_n^2 + \alpha_1 s_n + 1}} \cdot \underset{\text{another second order section}}{\frac{A_{o2}}{s_n^2 + \alpha_2 s_n + 1}} \cdot \underset{\text{first order section}}{\frac{A_o}{s_n + 1}}$$

Each term in the denominator has its own damping coefficient and critical frequency. Table 7.1 shows the denominator polynomials upto 8-th order Butterworth filter (see Appendix 7.1).

Table 7.1 Normalized Butterworth polynomial

Order n	Factors of polynomials
1.	$s_n + 1$
2.	$s_n^2 + 1.414\ s_n + 1$
3.	$(s_n + 1)\ (s_n^2 + s_n + 1)$
4.	$(s_n^2 + 0.765\ s_n + 1)\ (s_n^2 + 1.848\ s_n + 1)$
5.	$(s_n + 1)\ (s_n^2 + 0.618\ s_n + 1)\ (s_n^2 + 1.618\ s_n + 1)$
6.	$(s_n^2 + 0.518\ s_n + 1)\ (s_n^2 + 1.414\ s_n + 1)\ (s_n^2 + 1.932\ s_n + 1)$
7.	$(s_n + 1)\ (s_n^2 + 0.445\ s_n + 1)\ (s_n^2 + 1.247\ s_n + 1)\ (s_n^2 + 1.802\ s_n + 1)$
8.	$(s_n^2 + 0.390\ s_n + 1)\ (s_n^2 + 1.111\ s_n + 1)\ (s_n^2 + 1.663\ s_n + 1)\ (s_n^2 + 1.962\ s_n + 1)$

Example 7.1

Design a second order Butterworth low-pass filter having upper cut-off frequency 1 kHz. Then determine its frequency response.

Solution

Given f_h = 1 kHz = 1/2 π RC. Let C = 0.1 μF, gives the choice of R = 1.6 kΩ. From Table 7.1, for n = 2, the damping factor α = 1.414. Then the pass band gain $A_o = 3 - \alpha = 3 - 1.414 =$ 1.586. The transfer function of the normalised second order low-pass Butterworth filter is

$$\frac{1.586}{s_n^2 + 1.414\ s_n + 1}$$

Now $A_o = 1 + R_F/R_i = 1.586 = 1 + 0.586$. Let R_F = 5.86 kΩ and R_i = 10 kΩ. Then we get A_o = 1.586. The circuit realized is as in Fig. 7.4 with component values as R = 1.6 kΩ, C = 0.1 μF, R_F = 5.86 kΩ and R_i = 10 kΩ.

For minimum dc offset $R_i || R_F = 2R$ (at dc condition, capacitors are open) which has not been taken into consideration here, otherwise, we would have to modify the values of R and C accordingly which comes out to be R = 1.85 kΩ, C = 0.086 μF, R_F = 5.86 kΩ, R_i = 10 kΩ.

The frequency response data is shown in Table 7.2 using Eq. 7.25 and the frequency range is taken from 0.1 f_h to 10 f_h i.e., 100 Hz to 10 kHz as f_h = 1 kHz.

Table 7.2

Frequency, f in Hz	Gain magnitude in dB 20 log (v_o/v_i)
100 (0.1 f_h)	4.00
200 (0.2 f_h)	4.00
500 (0.5 f_h)	3.74
1000 (1.0 f_h)	1.00
5000 (5 f_h)	−23.95
10000 (10 f_h)	−35.99

Example 7.2

Design a fourth order Butterworth low-pass filter having upper cut-off frequency 1 kHz.

Solution

The upper cut-off frequency, f_h = 1 kHz = $1/2\pi RC$. Let C = 0.1 μF gives the choice of R = 1.6 kΩ. From Table 7.1, for n = 4, we get two damping factors namely, α_1 = 0.765 and α_2 = 1.848. Then the pass band gain of two quadratic factors are

$$A_{o1} = 3 - \alpha_1 = 3 - 0.765 = 2.235$$

$$A_{o2} = 3 - \alpha_2 = 3 - 1.848 = 1.152$$

The transfer function of fourth order low-pass Butterworth filter is

$$\frac{2.235}{s_n^2 + 0.765 s_n + 1} \cdot \frac{1.152}{s_n^2 + 1.848 s_n + 1}$$

Now, $$A_{o1} = 1 + \frac{R_{F1}}{R_{i1}} = 2.235 = (1 + 1.235)$$

Let R_{F1} = 12.35 kΩ and R_{i1} = 10 kΩ, then we get A_{o1} = 2.235
Similarly,

$$A_{o2} = 1.152 = 1 + 0.152 = 1 + \frac{R_{F2}}{R_{i2}}$$

Let R_{F2} = 15.2 kΩ and R_{i2} = 100 kΩ, which gives A_{o2} = 1.152.
The circuit realization is shown in Fig. 7.7.

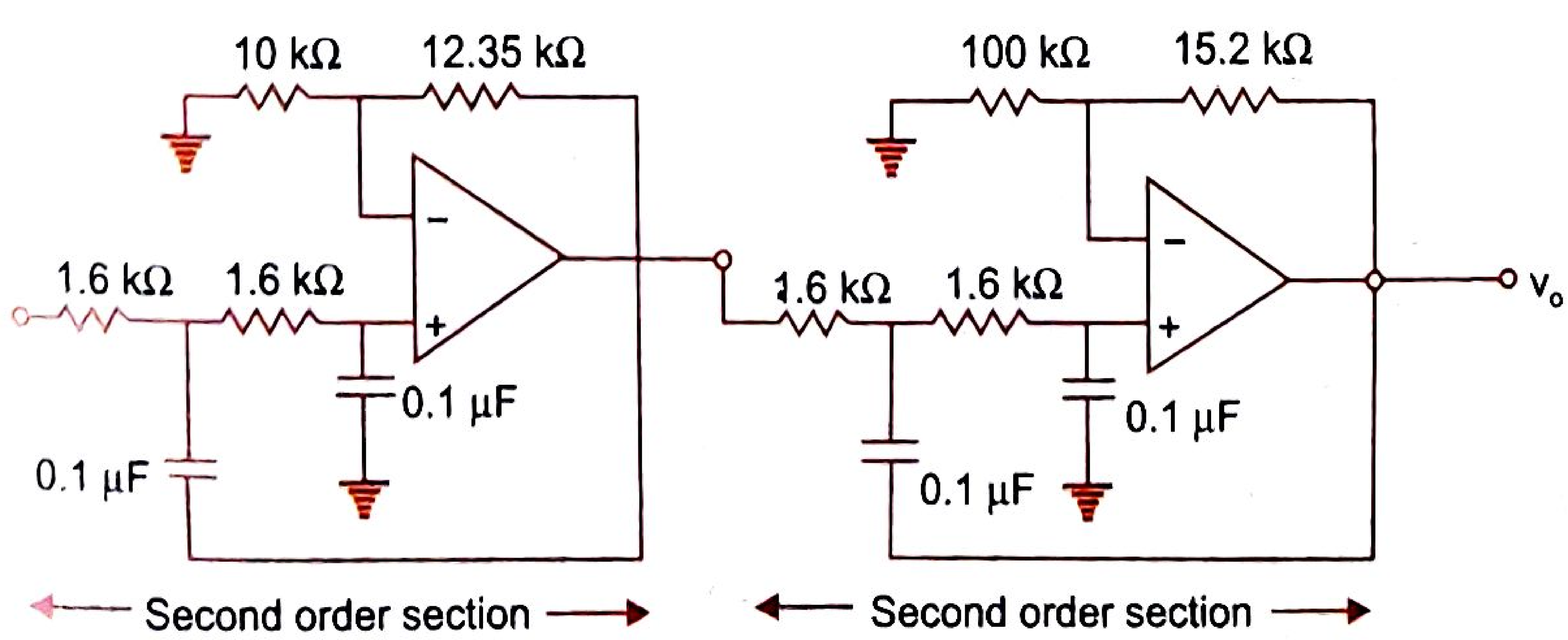


Fig. 7.7 Realization of 4-th order Butterworth low-pass filter

Example 7.3

Determine the order of a low-pass Butterworth filter that is to provide 40 dB attenuation at $\omega_h = 2$.

Solution

Use Eq. 7.26, then

$$20 \log \frac{H(j\omega)}{A_o} = -40 \text{ dB}$$

gives

$$\frac{H(j\omega)}{A_o} = 0.01$$

so

$$(0.01)^2 = \frac{1}{1+2^{2n}}$$

or,

$$2^{2n} = 10^4 - 1$$

Solving for n, we get $n = 6.64$

Since the order of the filter must be an integer so, $n = 7$.

7.2.4 High Pass Active Filter

A high pass filter is the complement of the low pass filter and can be obtained simply by interchanging R and C in the low pass configurations discussed earlier. A first order high pass filter is shown in Fig 7.8. It can be seen that a single RC network is connected to the non-inverting terminal of the op-amp and the gain of the filter is controlled by the feedback network $R_i R_F$.

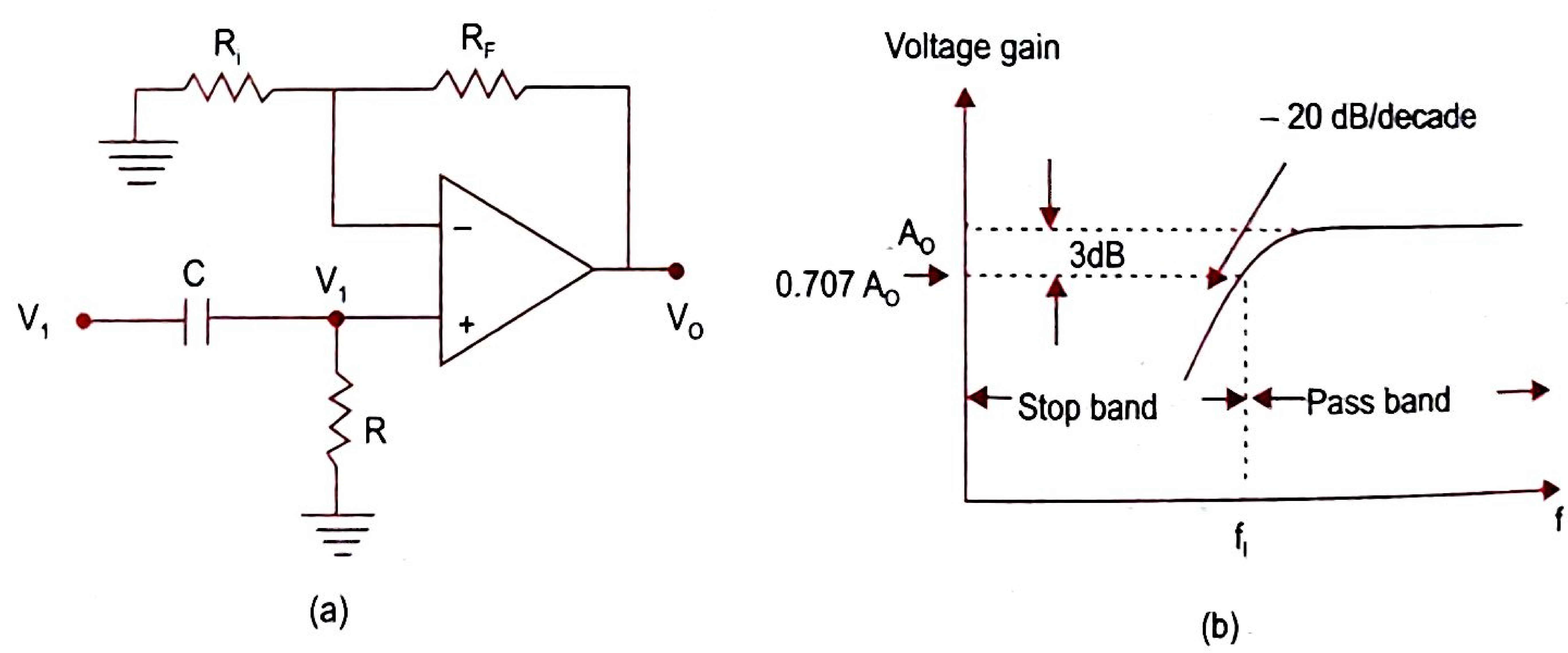


Fig 7.8 (a) First order high-pass filter. (b) Frequency response

The voltage V_1 at the non-inverting terminal is given by

$$V_1(s) = \frac{R}{R+\frac{1}{sC}} V_i(s) \tag{7.27}$$

or

$$\frac{V_1(s)}{V_i(s)} = \frac{RCs}{1+RCs} \tag{7.28}$$

where $V(s)$ is the Laplace transform of v in time domain.

Further, output voltage V_o is given by

$$V_o(s) = \left(1+\frac{R_F}{R_i}\right)V_i(s) \tag{7.29}$$

The closed loop gain of the op-amp is,

$$A_o = \frac{V_o(s)}{V_1(s)} = 1+\frac{R_F}{R_C} \tag{7.30}$$

Thus, the overall transfer function is obtained as

$$H(S) = \frac{V_o(s)}{V_1(s)} = \frac{A_o \times RCs}{1+RCs} \tag{7.31}$$

or

$$H_{HP}(j\omega) = \frac{A_o j f/f_l}{1+j(f/f_l)} \quad [s = j\omega] \tag{7.32}$$

where,

$$f_l = \frac{1}{2\pi RC}$$

The frequency response of the filter is obtained from the magnitude, that is

$$|H_{HP}(jf)| = \left|\frac{V_o}{V_i}\right| = \frac{A_o f/f_l}{\sqrt{1+(f/f_l)^2}} \tag{7.33}$$

$$= \frac{A_o}{\sqrt{1+(f_l/f)^2}} \tag{7.34}$$

The frequency response of the first order high-pass filter is shown in Fig 7.8(b). At very high frequencies ie. $f > f_l$, the gain is constant at A_o, and for $f < f_l$, the gain rolls-off at a rate of –20 dB/decade. The frequency range below f_l is called the stop band and the frequency range above f_l is called the pass-band.

It may be noted that the high-pass filter can be obtained from the low pass filter by applying the transformation

$$\left.\frac{s}{\omega_o}\right|_{LP} = \left.\frac{\omega_o}{s}\right|_{HP} \tag{7.35}$$

Thus, a low pass filter can be converted to a high pass filter simply by interchanging R and C

Example 7.4

Design and plot the frequency response of a first order high pass filter for pass band gain of 2 and lower cut-off frequency of 2KHZ.

Solution

Given $f_l = 2$ kHz

Assume $C = 0.01\ \mu F$

since $f_l = \dfrac{1}{2\pi RC}$

Therefore, $R = \dfrac{1}{2\pi f_l C}$

$$= \frac{1}{2\pi \times 2 \times 10^3 \times 10^{-8}}$$

$= 7.95$ kHZ

Further, to obtain pass band gain of 2,

$$A_o = 2 = 1 + \frac{R_F}{R_i}$$

Assume $R_F = R_i = 10\ k\Omega$

Thus, we can obtain pass band gain of 2. The frequency response data is shown in Table 7.3.

Table 7.3

Frequency (*Hz*)	*Gain* $\lvert V_o/V_i \rvert$	*Gain in dB* *20 log* $\dfrac{V_o}{V_i}$
100	0.10	- 20.01
200	0.20	- 14.02
400	0.39	- 8.13
1000	0.89	- 0.97
3000	1.66	4.42
10,000	1.96	5.85
100 kHz	2.00	6.02

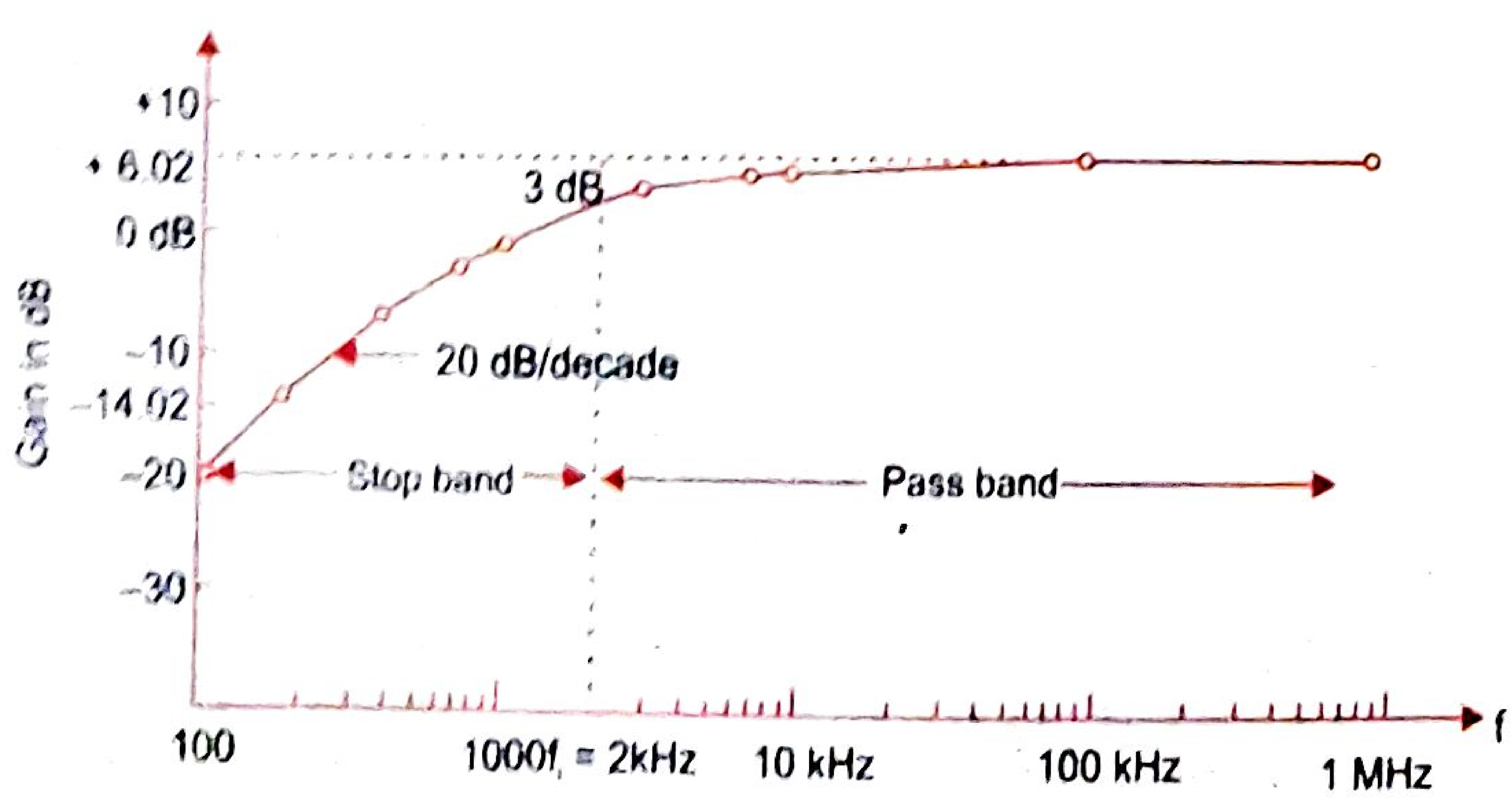


Fig. 7.9 Frequency response of first order high-pass filter.

Second Order High Pass Active Filter

High pass filter is the complement of the low pass filter and can be obtained simply by interchanging R and C in the low pass configuration and is shown in Fig. 7.10. Putting $Y_1 = Y_2 = sC$ and $Y_3 = Y_4 = G = 1/R$ in the general Eq. (7.17), the transfer function becomes.

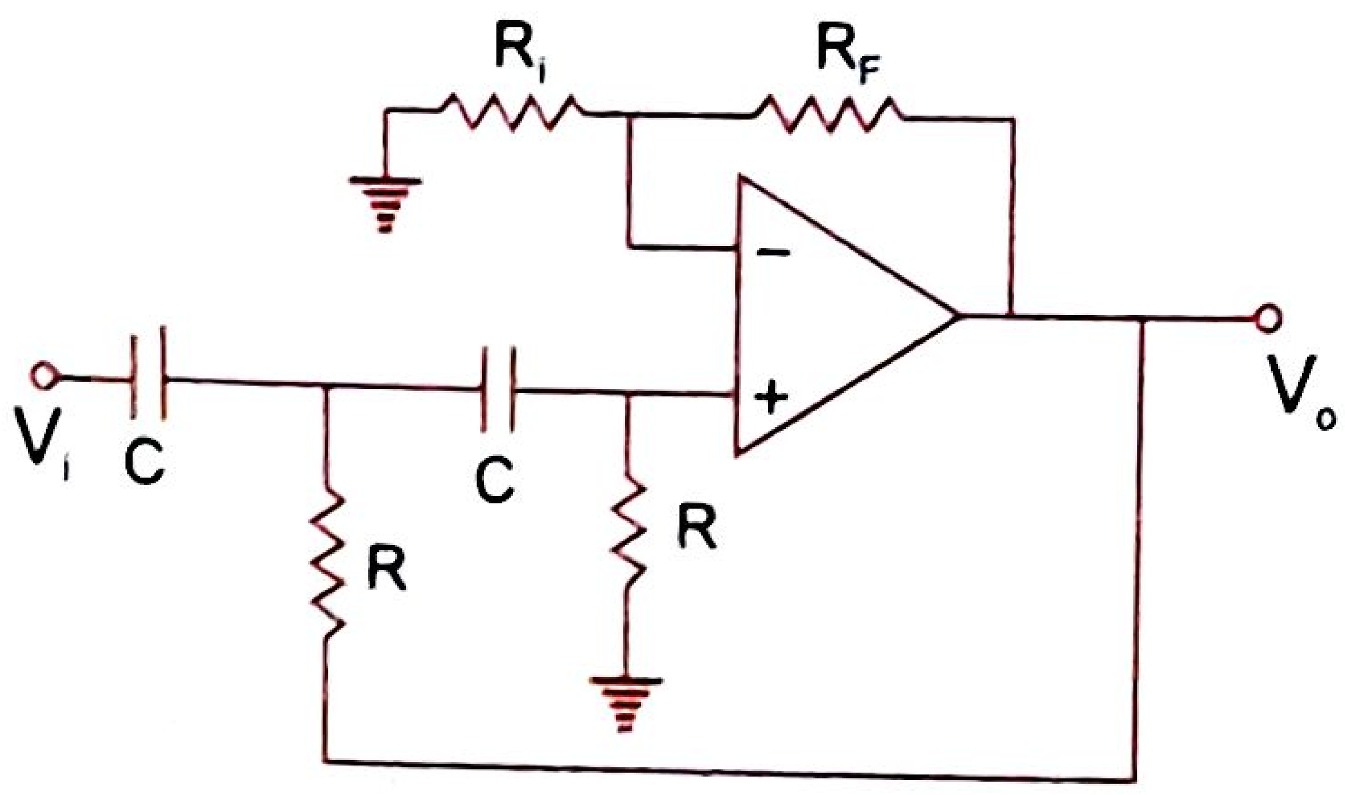


Fig. 7.10 Second order high pass filter

$$H_{HP}(s) = \frac{A_o s^2}{s^2 + (3 - A_o)\omega_l s + \omega_l^2} \quad (7.36)$$

where $$\omega_l = \frac{1}{RC}$$

or, $$H_{HP}(s) = \frac{A_o}{1 + \frac{\omega_l}{s}(3 - A_o) + \left(\frac{\omega_l}{s}\right)^2} \quad (7.37)$$

From Eq. (7.37), for $\omega = 0$, we get $H = 0$ and for $\omega = \infty$, we get $H_{HP} = A_o$. So the circuit indeed acts like high pass filter. The lower cut-off frequency

$$f_l = f_{3dB} = \frac{1}{2\pi RC}$$

and is same as in the low pass filter.

Putting $s = j\omega$ in Eq. (7.37) and $3 - A_o = \alpha = 1.414$, the voltage gain magnitude equation of the second order Butterworth high pass filter can be obtained as

$$|H_{HP}(j\omega)| = \left|\frac{V_o}{V_i}\right| = \frac{A_o}{\sqrt{1 + (f_l/f)^4}} \quad (7.38)$$

Hence

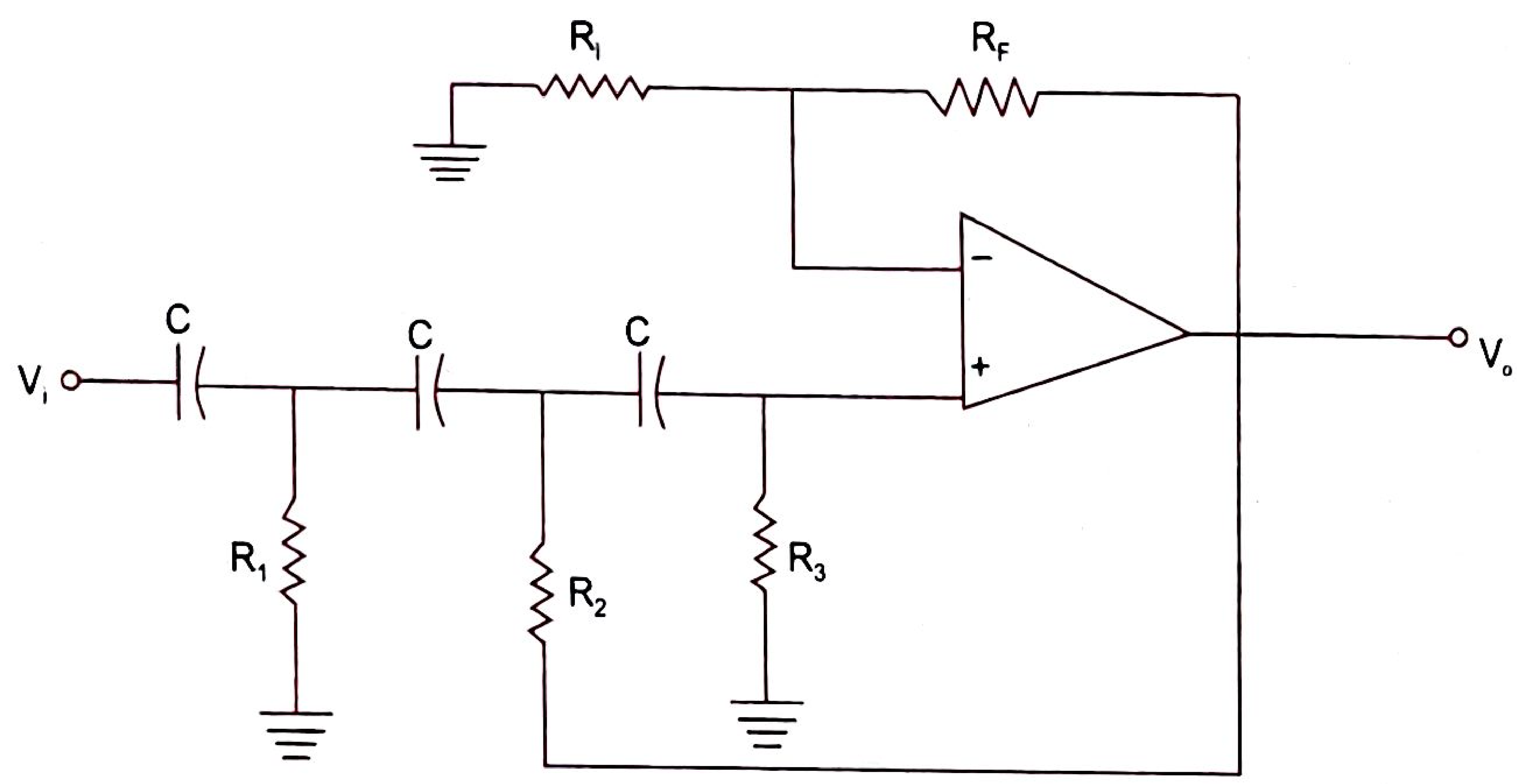
$$\left|\frac{H_{HP}(j\omega)}{A_o}\right| = \frac{1}{\sqrt{1 + \left(\frac{f_l}{f}\right)^4}} \quad (7.39)$$

Higher-order High-pass Filters

The magnitude of the voltage transfer function for the nth order Butterworth high-pass filter is given by

$$|H_{HP}(jf)| = \frac{1}{\sqrt{1 + \left(\frac{f_l}{f}\right)^{2n}}} \quad (7.40)$$

Higher order filters can be designed by adding additional RC networks, that is by cascading a required number of first and second order filters. Fig 7.11 (a) and (b) shows a third-order and fourth order high pass Butterworth filter respectively.



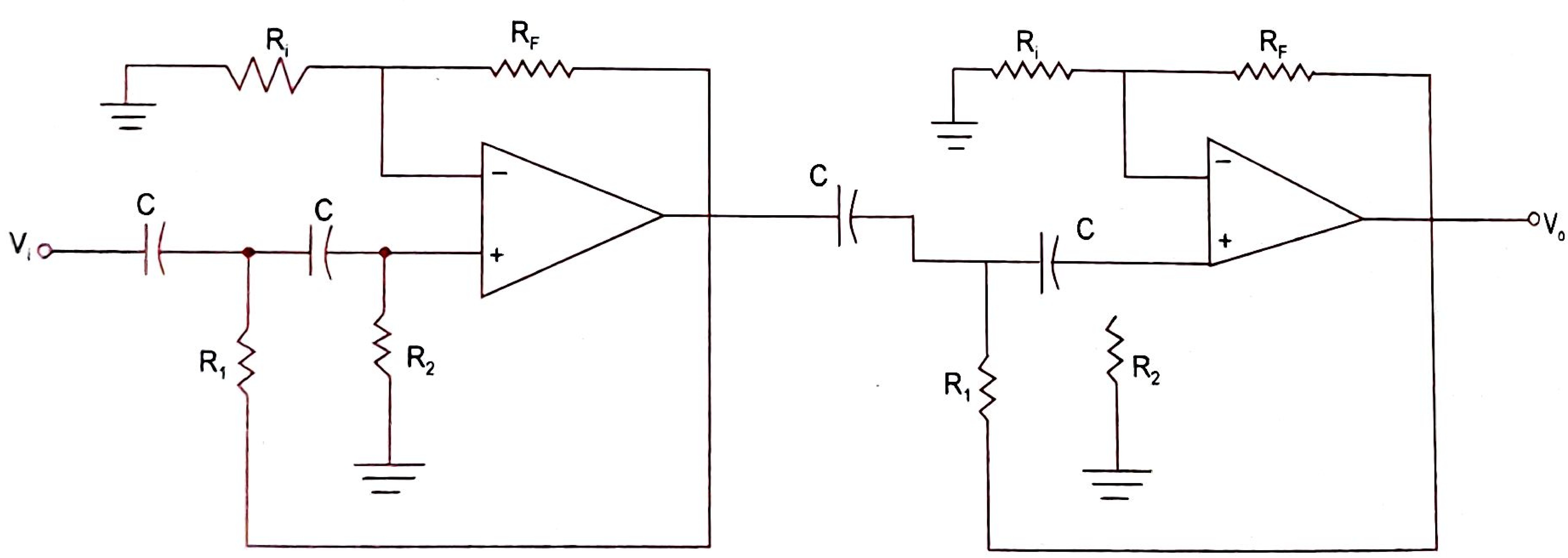


Fig 7.11 (a) Third order

(b) Fourth order High pass Filter

Example 7.5

Design a second order Butterworth high pass filter having lower cut-off frequency of 1 kHz.

Solution

Refer to Example 7.1. The same value of cut-off frequency for Butterworth LPF has been taken so the values of R and C will be the same. Also the values of R_F and R_i are same as calculated in Example 7.1. Only the frequency response will have to be calculated using Equation. (7.34). The circuit configuration is as in Fig. 7.8 with component values R = 1.6. kΩ, C = 0.1 μF, R_F = 5.86 kΩ, R_i = 10 kΩ.

7.2.5 Band Pass Filter

There are two types of band pass filters which are classified as per the figure of merit or quality factor Q.

(i) Narrow band pass filter ($Q > 10$)
(ii) Wide band pass filter ($Q < 10$)

The following relationships are important:

$$Q = f_o/BW = f_o/(f_h - f_l)$$

and

$$f_o = \sqrt{f_h f_l}$$

where f_h = upper cut-off frequency
f_l = lower cut-off frequency
f_o = the central frequency

Narrow Band Pass Filter

The important parameters in a band pass filter (BPF) are upper and lower cut-off frequencies (f_h and f_l), the band width (BW), the central frequency (f_o), the central frequency gain A_o and selectivity Q. Consider the circuit of Fig. 7.12 (a). The circuit has two feedback paths and the op-amp is used in inverting mode of operation.

The node voltage equation at node A is

$$v_i Y_1 + v_o Y_3 = v_A (Y_1 + Y_2 + Y_3 + Y_4) \tag{7.41}$$

Assuming, $v_B = 0$ (virtual ground), the node voltage equation at node B is,

$$v_A Y_2 = -v_o Y_5$$

$$v_A = -v_o (Y_5/Y_2) \tag{7.42}$$

Putting v_A in Eq. (7.41), we get

$$v_i Y_1 + v_o Y_3 = -\frac{v_o Y_5 (Y_1 + Y_2 + Y_3 + Y_4)}{Y_2}$$

or,

$$v_i Y_1 = v_o \left[-\frac{Y_2 Y_3 + Y_1 Y_5 + Y_2 Y_5 + Y_3 Y_5 + Y_4 Y_5}{Y_2} \right]$$

Hence

$$\frac{v_o}{v_i} = -\frac{Y_1 Y_2}{Y_2 Y_3 + Y_1 Y_5 + Y_2 Y_5 + Y_3 Y_5 + Y_4 Y_5} \tag{7.43}$$

For this circuit to be band pass filter, put $Y_1 = G_1$, $Y_2 = sC_2$, $Y_3 = sC_3$, $Y_4 = G_4$ and $Y_5 = G_5$ as shown in Fig. 7.12 (b). Then the transfer function becomes,

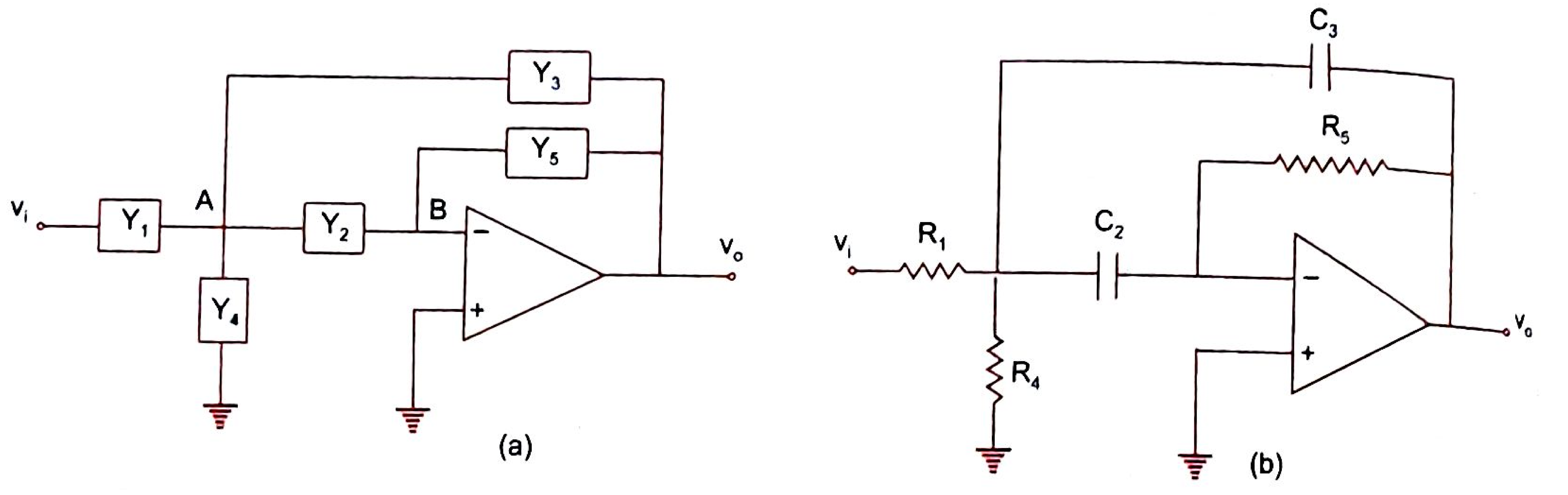


Fig. 7.12 (a) Band-pass configuration (b) Second order band-pass filter

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{-sG_1C_2}{s^2C_2C_3 + s(C_2+C_3)G_5 + G_5(G_1+G_4)}$$

or, $$H(s) = \frac{-G_1}{sC_3 + G_5(C_2+C_3)/C_2 + (G_1+G_4)G_5/sC_2} \tag{7.44}$$

The transfer function of Eq. (7.44) is equivalent to the gain expression of a parallel RLC circuit of Fig. 7.13 (a) driven by a current source $G'v_i$ and with band pass characteristics as shown in Fig. 7.13 (b). The gain expression is,

$$\frac{V_o(s)}{V_i(s)} = -\frac{G'}{Y} = \frac{-G'}{sC + G + 1/sL} \tag{7.45}$$

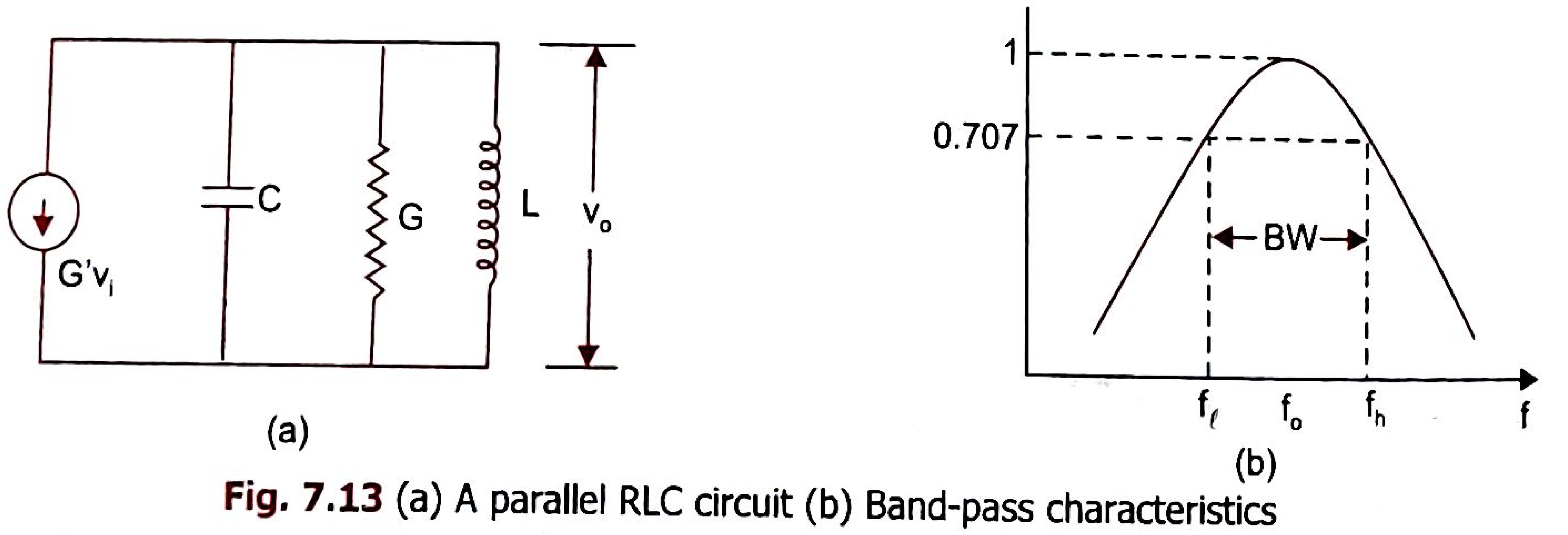


Fig. 7.13 (a) A parallel RLC circuit (b) Band-pass characteristics

Comparing the gain expression of Eqs. (7.44) and (7.45), we get,

$$G' = G_1 \tag{7.46}$$

$$L = \frac{C_2}{G_5(G_1+G_4)} \tag{7.47}$$

$$G = \frac{G_5(C_2+C_3)}{C_2} \tag{7.48}$$

and $$C = C_3 \tag{7.49}$$

At resonance, the circuit of Fig. 7.13 (a) has unity power factor, i.e. imaginary part is zero which gives the resonant frequency ω_o as,

$$\omega_o^2 = \frac{1}{LC} = G_5\frac{(G_1+G_4)}{C_2C_3} \tag{7.50}$$

The gain at resonance is,

$$\left.\frac{v_o}{v_i}\right|_{\omega=\omega_o} = -\frac{G'}{G} = -\frac{G_1}{G} = -\frac{(G_1/G_5)C_2}{C_2+C_3}$$

$$= -\frac{(R_5/R_1)C_2}{C_2+C_3} \tag{7.51}$$

The Q factor at resonance is,

$$Q_o = \frac{\omega_o L}{R} = \omega_o RC = \frac{\omega_o C}{G} = \frac{\omega_o C_2 C_3}{(C_2+C_3)G_5} \tag{7.52}$$

The bandwidth BW is given by,

$$BW = f_h - f_l = \frac{f_o}{Q_o} = \frac{\omega_o}{2\pi Q_o} = \frac{\omega_o}{2\pi R\omega_o C}$$

$$= \frac{1}{2\pi RC} = \frac{G}{2\pi C} = \frac{G_5(C_2+C_3)}{2\pi C_2 C_3} \tag{7.53}$$

and the centre frequency $f_o = \sqrt{f_h f_l}$

Now for $C_2 = C_3 = C$, the gain at resonant frequency from Eq. (7.51) is,

$$\left.\frac{v_o}{v_i}\right|_{\omega=\omega_o} = -\frac{R_5}{2R_1} = -A_o \tag{7.54}$$

$$\omega_o = \frac{\sqrt{G_5(G_1+G_4)}}{C} \tag{7.55}$$

$$BW = \frac{G_5}{\pi C} = \frac{1}{\pi R_5 C} \tag{7.56}$$

We have three independent design parameter Eqs. (7.54), (7.55) and (7.56) for gain at resonance, resonant frequency and bandwidth. But there are four unknown parameters of the circuit such as C, G_1, G_4 and G_5. So we have to choose any one parameter arbitrarily.

Figure 7.14 shows a plot of freq-uency response for different values of Q. The higher the Q, the sharper the filter. Below 0.5 f_o and above 2 f_o, all filters roll-off at – 20 dB/decade independent of the value of Q. This is limited by the two RC pairs in the circuit. To obtain sharper roll-off rate away from the center frequency, one should cascade several filters.

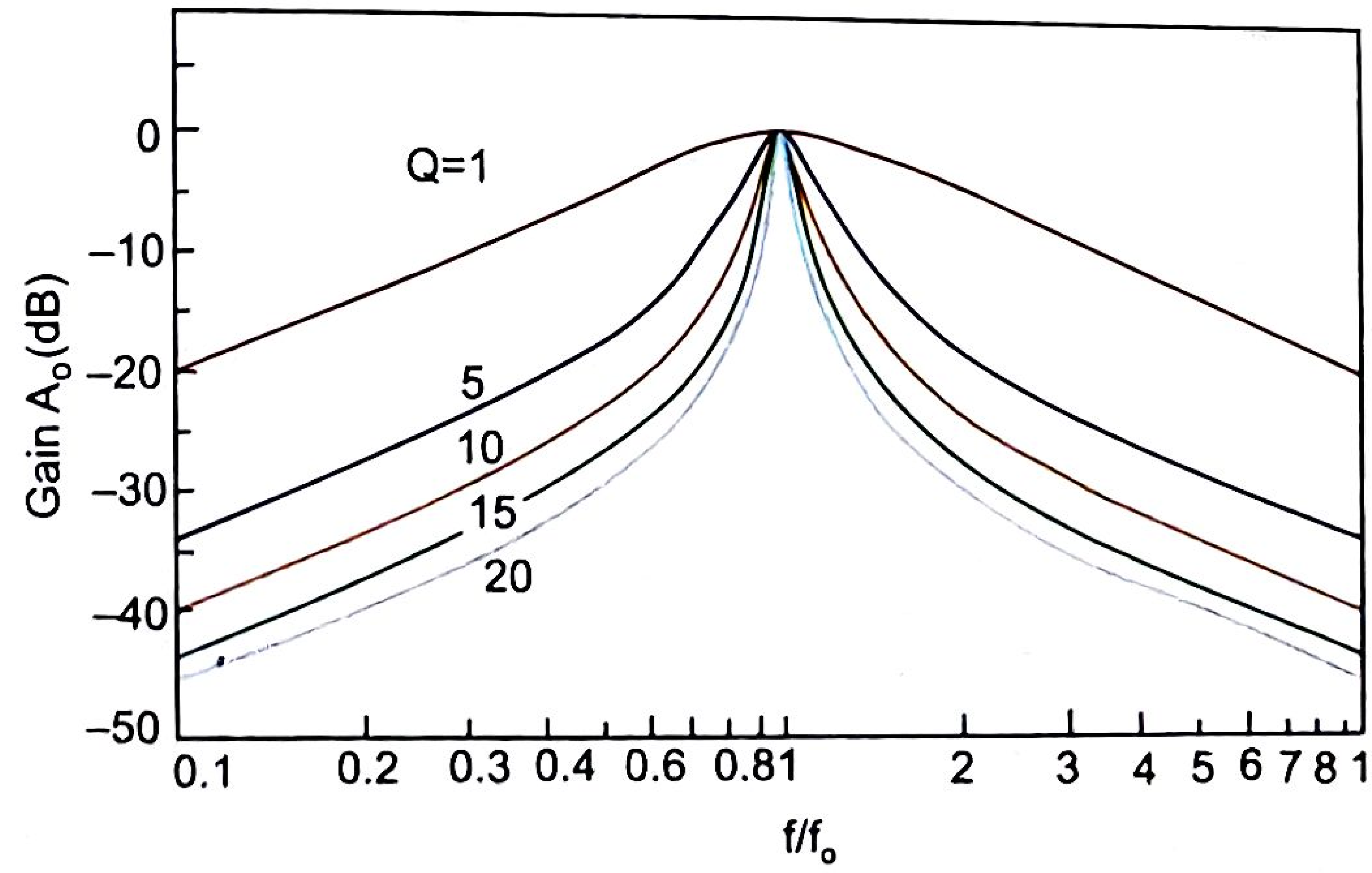


Fig. 7.14 Single op-amp band-pass filter response

It may further be noted that using Eqs. (7.52, 7.54 and 7.55) in Eq. (7.44), the standard transfer function of a bandpass filter is obtained as,

$$H(s) = \frac{-A_o(\omega_o/Q)s}{s^2 + (\omega_o/Q)s + \omega_o^2} = \frac{-A_o \alpha \omega_o s}{s^2 + \alpha\omega_o s + \omega_o^2} \tag{7.57}$$

or, in dB, we get, $20 \log|H(s)| = 20 \log\left|\frac{A_o \alpha \omega_o s}{s^2 + \alpha \omega_o s + \omega_o^2}\right|$ (7.58)

where the damping factor $\alpha = \frac{1}{Q}$.

It is obvious from Eq. (7.57) that for $\omega << \omega_o$ and $\omega >> \omega_o$, the gain is zero and for $\omega = \omega_o$ the gain is A_o. It may be noted that A_o is negative.

Wide Band-Pass Filter

A wide band-pass filter can be formed by cascading a HPF and LPF section. If the HPF and LPF are of the first order, then the band-pass filter (BPF) will have a roll-off rate of – 20 dB/decade.

For the high pass section of Fig. 7.15 the magnitude of gain is

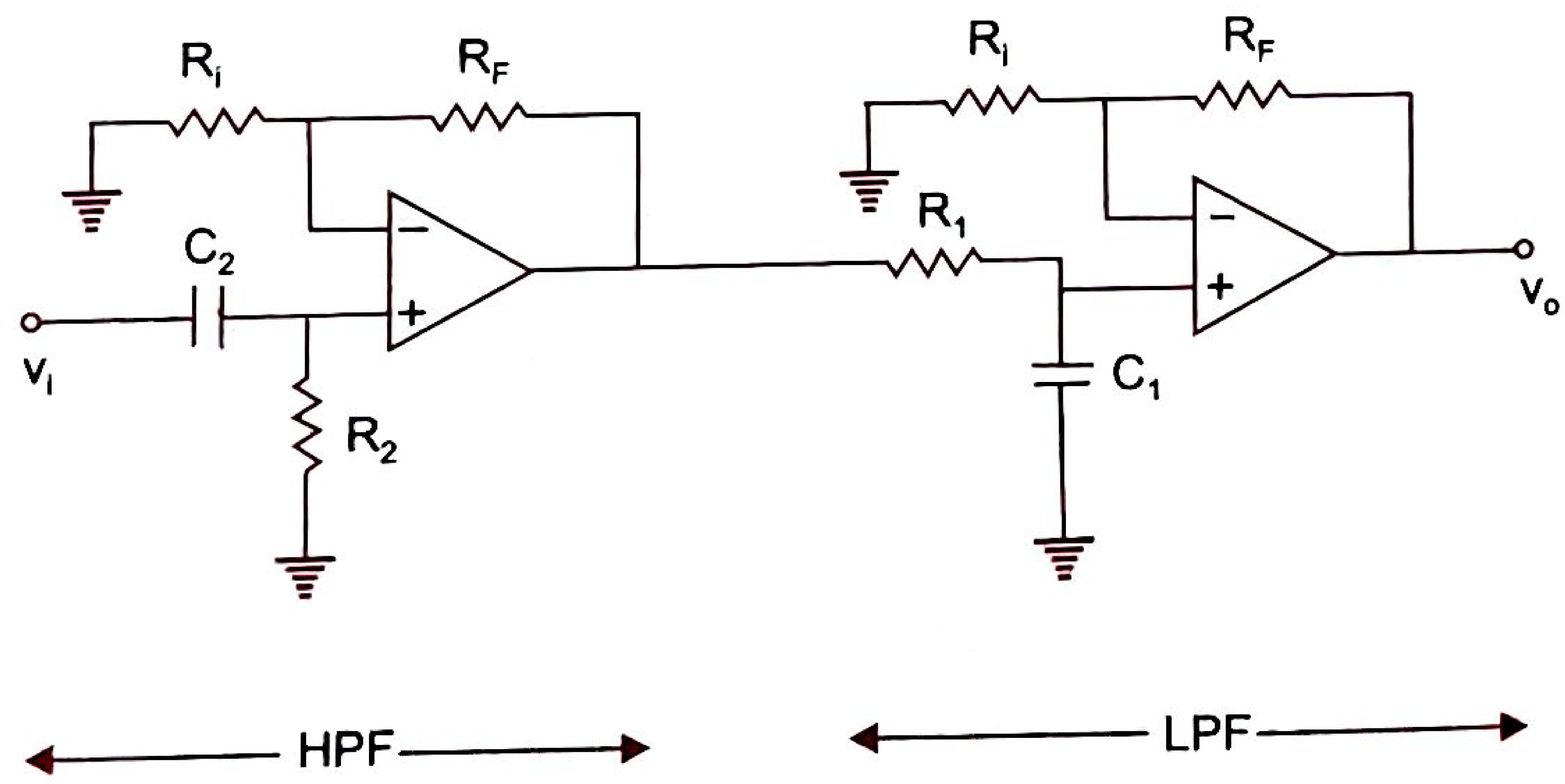


Fig. 7.15 First order band-pass filter

$$|H_{HP}| = \left|\left(1 + \frac{R_F}{R_i}\right)\frac{j2\pi f R_2 C_2}{1 + j2\pi f R_2 C_2}\right| = \left|A_{01}\frac{j(f/f_l)}{1 + j(f/f_l)}\right| = \frac{A_{01}(f/f_l)}{\sqrt{1 + (f/f_l)^2}} \tag{7.59}$$

where $$f_l = \frac{1}{2\pi R_2 C_2} \tag{7.60}$$

Similarly, for the low-pass section of Fig. 7.15, the magnitude of gain is

$$|H_{LP}| = \frac{A_{02}}{\sqrt{1 + (f/f_h)^2}} \tag{7.61}$$

where $$f_h = \frac{1}{2\pi R_1 C_1} \tag{7.62}$$

The voltage gain magnitude of the wide band pass filter is the product of that of LPF and HPF. One can calculate the frequency response from the equation

$$\left|\frac{v_o}{v_i}\right| = \left|\frac{A_o(f/f_l)}{\sqrt{\left[1+(f/f_l)^2\right]\left[1+(f/f_h)^2\right]}}\right| \tag{7.63}$$

where the total pass band gain $A_o = A_{o1} \times A_{o2}$.

In a similar fashion, to obtain BPF of –40 dB/decade fall-off rate, second order HPF and LPF sections are to be cascaded.

Example 7.6

Design a wide-band pass filter having f_l = 400 Hz, f_h = 2 kHz and pass band gain of 4. Find the value of Q of the filter.

Solution

The pass band gain is 4. The LPF and HPF sections each of Fig. 7.15 may be designed to give gain of 2, that is, $A_o = 1 + R_F/R_i = 2$. So R_F and R_i should be equal. Let $R_F = R_i = 10$ kΩ for each of LPF and HPF sections.

For LPF, f_h = 2 kHz = $1/2\pi R_1C_1$. Let C_1 = 0.01 μF gives R_1 = 7.9 kΩ. For HPF, f_l = 400 Hz = $1/2\pi R_2C_2$. Let C_2 = 0.01 μF gives R_2 = 39.8 kΩ.

Again $f_o = \sqrt{f_h f_l} = \sqrt{2000 \times 400} = 894.4$

$$Q = f_o/BW = f_o/(f_h - f_l) = 894.4/(2000 - 400) = 0.56$$

Obviously, for wide band pass filter, Q is very low, i.e., $Q < 10$.

Example 7.7

The resonant frequency f_o of a band-pass filter is 1 kHz and its BW is 3 kHz. Find (i) Q (ii) f_l and f_h.

Solution

We know that quality factor, Q is given by,

(i) $Q = \dfrac{f_o}{BW} = \dfrac{1\times10^3}{3\times10^3} = 0.33$

Since $Q < 10$, it is a wide band filter.

(ii) It can be shown that

$$f_l = \sqrt{\frac{BW^2}{4} + f_o^2} - \frac{B}{2}$$

and $f_h = f_l + BW,$

we find $f_l = 302.77$ Hz

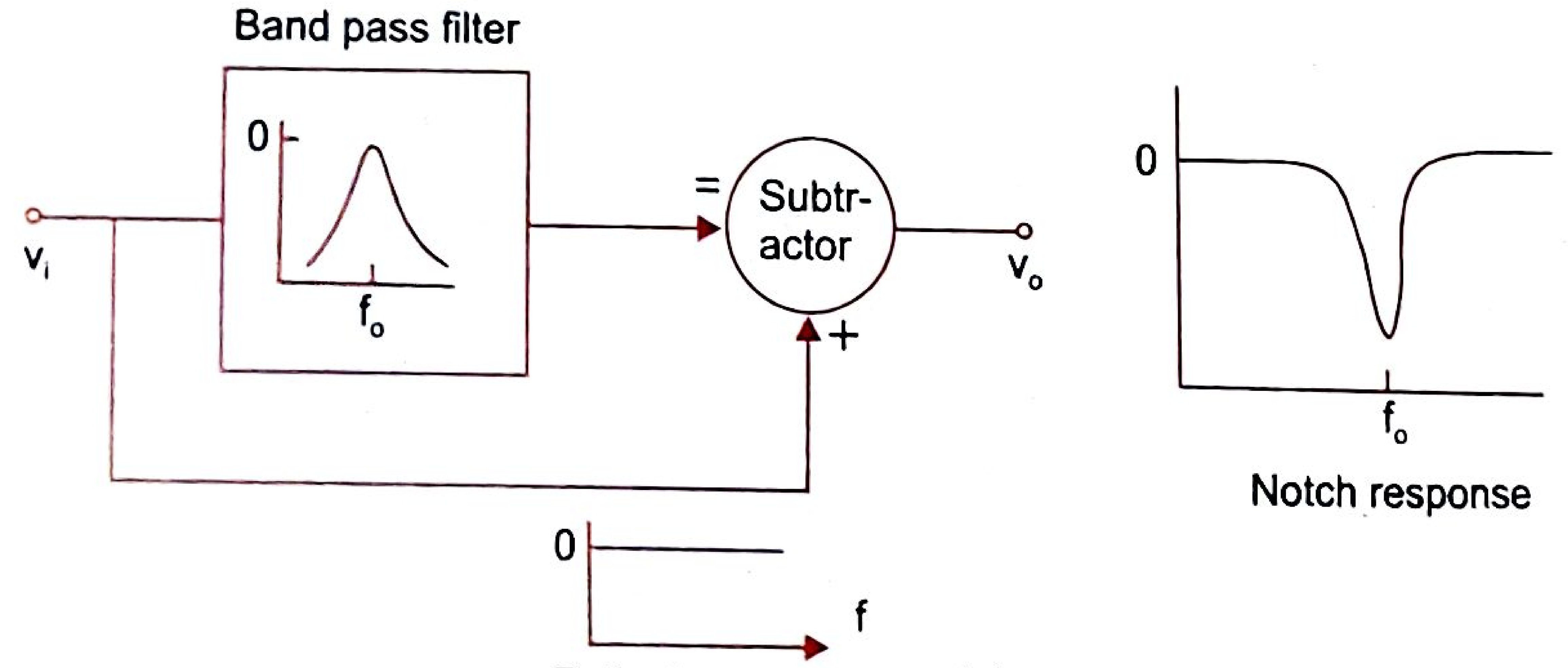
and $f_h = 3302.77$ Hz

7.2.6 Band Reject Filter

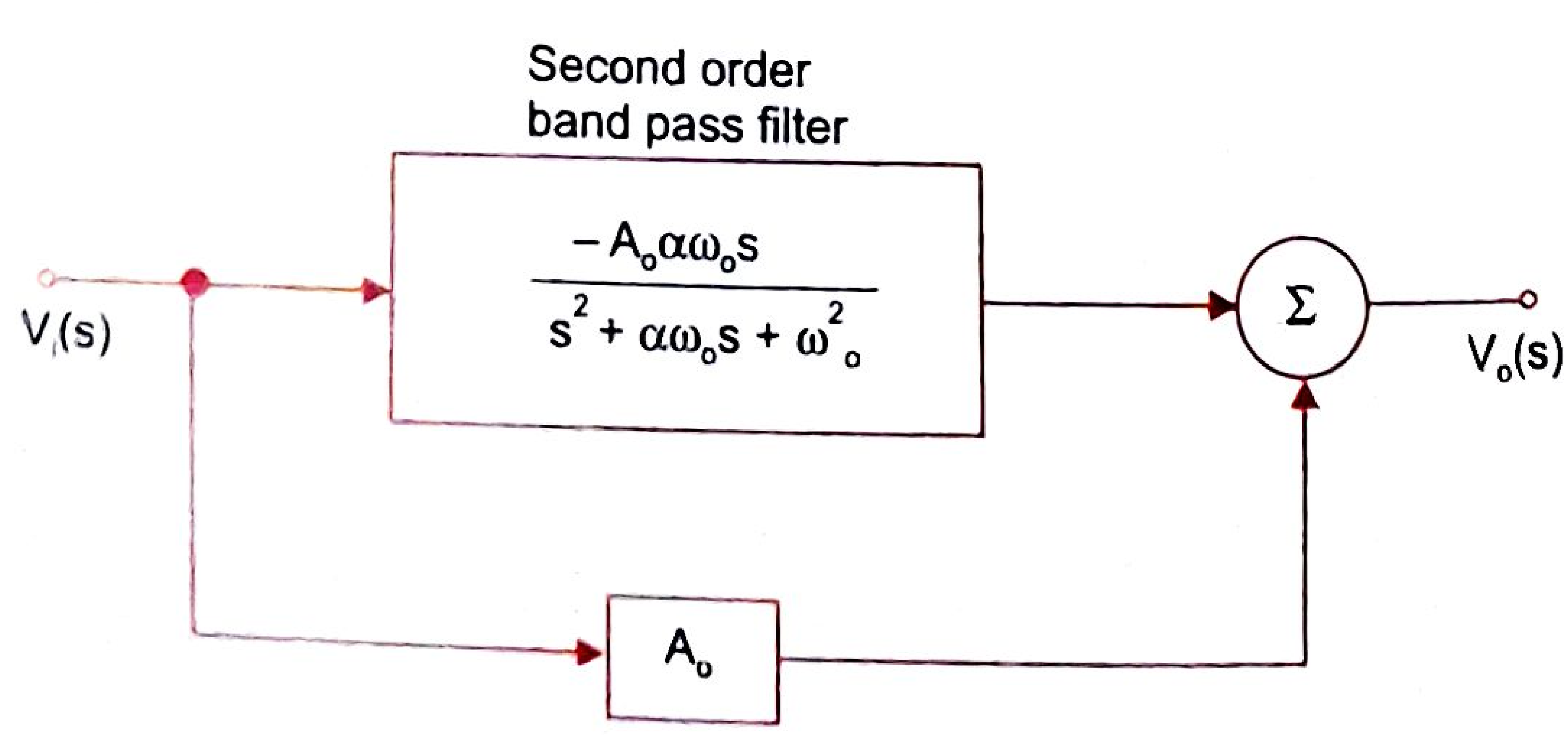
A band reject filter (also called a band stop or band elimination) can be either (i) Narrow band reject filter or (ii) Wide band reject filter. The narrow band reject filter is commonly called a ***notch filter*** and is useful for the rejection of a single frequency, such as 50 Hz power line frequency hum.

There are several ways to make notch filters. One simple technique is to subtract the band pass filter output from its input. This principle is illustrated in Fig. 7.13 (a).

The band pass filter discussed earlier has an inverted output as the gain or transfer Eq. (7.35) is negative. Therefore, while implementing Fig. 7.13 (a), we must use a summer instead of a subtractor. Also, the band pass filter has a gain of A_o, so that output at the centre frequency will be $-A_o \times v_i$. To completely subtract this output, the input of the summer must be precisely $A_o v_i$. Thus, a gain of A_o must be added between the input signal and the summer as shown in Fig. 7.13 (b). The output, of the circuit in the s domain is,



(a)



(b)

Fig. 7.16 (a) Notch filter block diagram (b) Practical notch filter block diagram

$$V_o(s) = A_o V_i(s) + \left(\frac{-A_o\, \alpha\, \omega_o s\, V_i(s)}{s^2 + \alpha\, \omega_o s + \omega_o^2}\right) \tag{7.64}$$

$$\frac{V_o(s)}{V_i(s)} = A_o - \frac{A_o\, \alpha\, \omega_o s}{s^2 + \alpha \omega_o s + \omega_o^2}$$

$$= A_o\left(1 - \frac{\alpha \omega_o s}{s^2 + \alpha\, \omega_o s + \omega_o^2}\right)$$

$$= \frac{A_o(s^2 + \omega_o^2)}{s^2 + \alpha \omega_o s + \omega_o^2} \tag{7.65}$$

This is the transfer function for a second order notch filter and the circuit schematic is shown in Fig. 7.17. It is evident from Eq. (7.65), that for $\omega << \omega_o$ and for $\omega >> \omega_o$ the pass band gain is $|A_o|$ and at frequency $\omega = \omega_o$ the gain is zero.

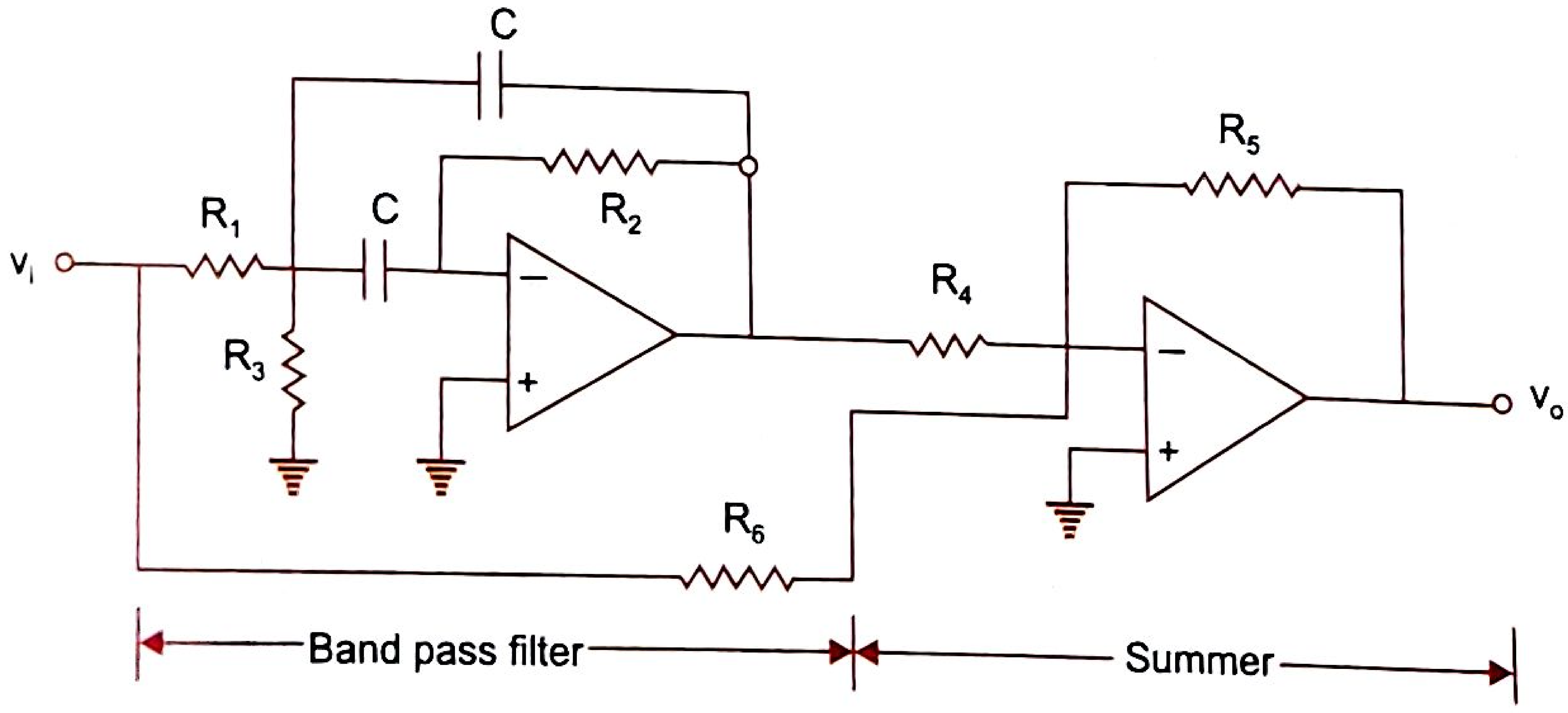


Fig. 7.17 Notch filter schematic

Another commonly used notch filter is the twin-T network as shown in Fig. 7.18 (a). We will determine the notch frequency, Q factor and bandwidth for this configuration.

Node voltage equations in s-domain (by KCL) for the active filter circuit of Fig. 7.18 (a) can be written as,

At node A: $\quad (V_i - V_A)\, sC + (V_o - V_A)\, sC + (KV_o - V_A)2G = 0$

or,

$$sC\, V_i + (sC + 2KG)\, V_o = 2\,(sC + G)\, V_A \tag{7.66}$$

where V_A is the Laplace transform of the voltage at node A. Similarly V_i and V_o are transformed input and output. The s in the parenthesis has been dropped in the Laplace transform for simplicity.

At node B: $\quad (V_i - V_B)G + (V_o - V_B)G + 2\,(KV_o - V_B)sC = 0$

or,

$$GV_i + (G + 2\,KsC)\, V_o = 2(G + sC)V_B \tag{7.67}$$

where V_B is the Laplace transform of the voltage at node B.

At node P: $\quad (V_A - V_o)\, sC + (V_B - V_o)G = 0$

or,

$$sC\, V_A + GV_B = (G + sC)V_o \tag{7.68}$$

where, $K = R_2/(R_1 + R_2)$ and $G = 1/R$

From these node voltage equations, the transfer function can be written as,

$$H(s) = \frac{V_o}{V_i} = \frac{G^2 + s^2C^2}{G^2 + s^2C^2 + 4(1-K)s\,CG}$$

$$= \frac{s^2 + (G/C)^2}{s^2 + (G/C)^2 + 4(1-K)s\,(G/C)} \quad (7.69)$$

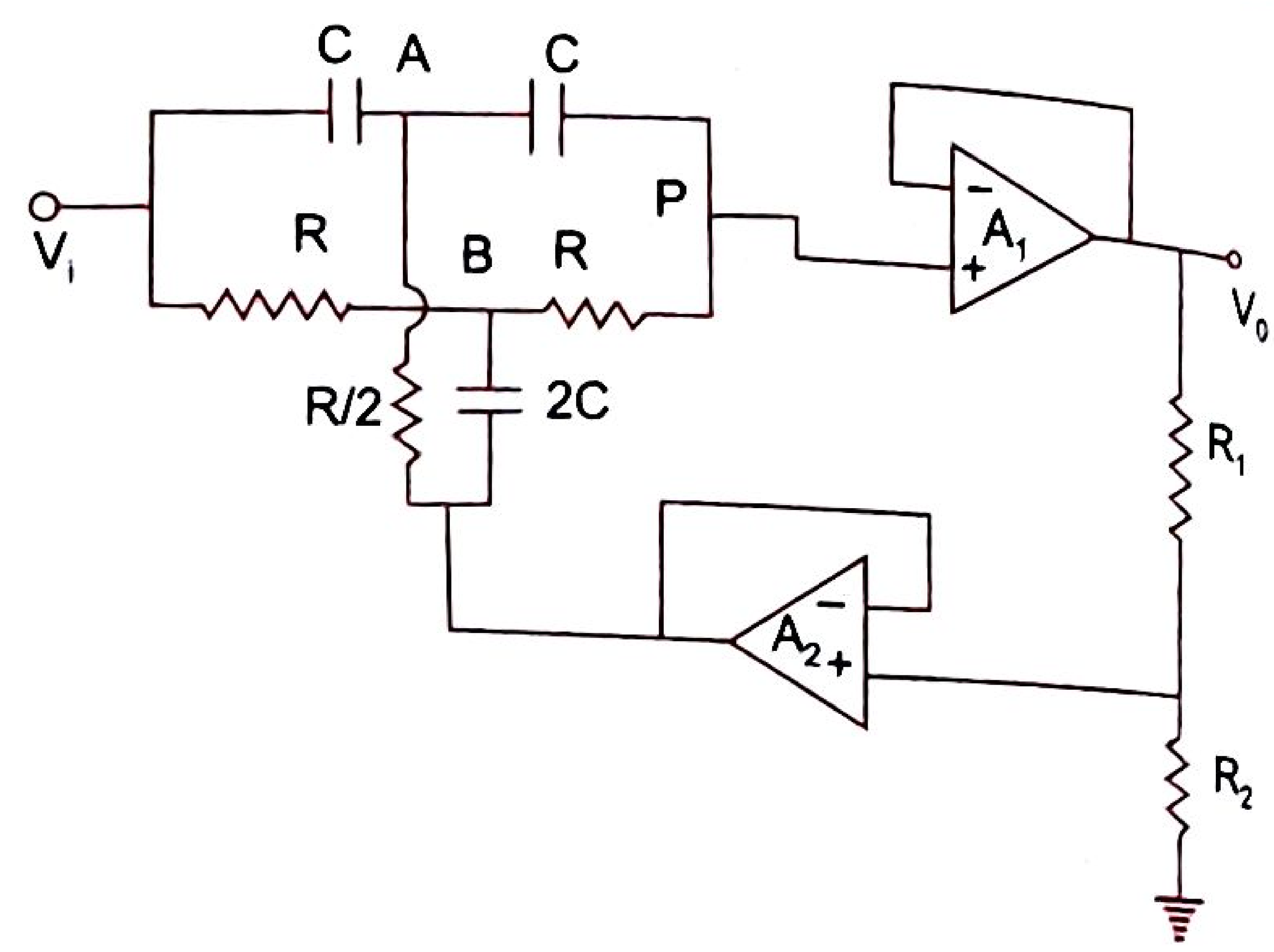


Fig. 7.18 (a) Twin-T notch filter

In the steady state (i.e. $s = j\omega$),

$$H(j\omega) = \frac{\omega^2 - \omega_o^2}{\omega^2 - \omega_o^2 - j4(1-K)\omega\omega_o} \quad (7.70)$$

where, $\omega_o = G/C = 1/RC$

i.e., $$f_o = \frac{1}{2\pi RC} \quad (7.71)$$

From Eq. (7.70), $H(j\omega)$ becomes zero for $\omega = \omega_o$ and approaches unity as $\omega << \omega_o$ and for $\omega >> \omega_o$. In practice, the high frequency response will be limited by the high frequency response of the op-amp. At 3-dB points, $|H| = 1/\sqrt{2}$

i.e. $$\omega^2 - \omega_o^2 = \pm 4(1-K)\,\omega\,\omega_o$$

or $$(\omega/\omega_o)^2 \pm 4(1-K)\,(\omega/\omega_o) - 1 = 0 \quad (7.72)$$

Solving the quadratic equation, we get the upper and lower half power frequencies as,

$$f_h = f_o\left[\sqrt{1 + 4(1-K)^2} + 2(1-K)\right] \quad (7.73)$$

and $$f_l = f_o\left[\sqrt{1 + 4(1-K)^2} - 2(1-K)\right] \quad (7.74)$$

The 3-dB bandwidth,

$$BW = f_h - f_l = 4(1-K)f_o \quad (7.75)$$

$$Q = \frac{f_o}{BW} = \frac{1}{4(1-K)} \quad (7.76)$$

As K approaches unity, Q factor becomes very large and BW approaches 0. In fact, mismatches between resistors and capacitors limit the Q-factor and BW to practically realizable value. It is advisable to use the components of 0.1 per cent tolerance resistors and 1 per cent tole-rance capacitors for very high value of Q-factor. The frequency response is shown in Fig. 7.18 (b).

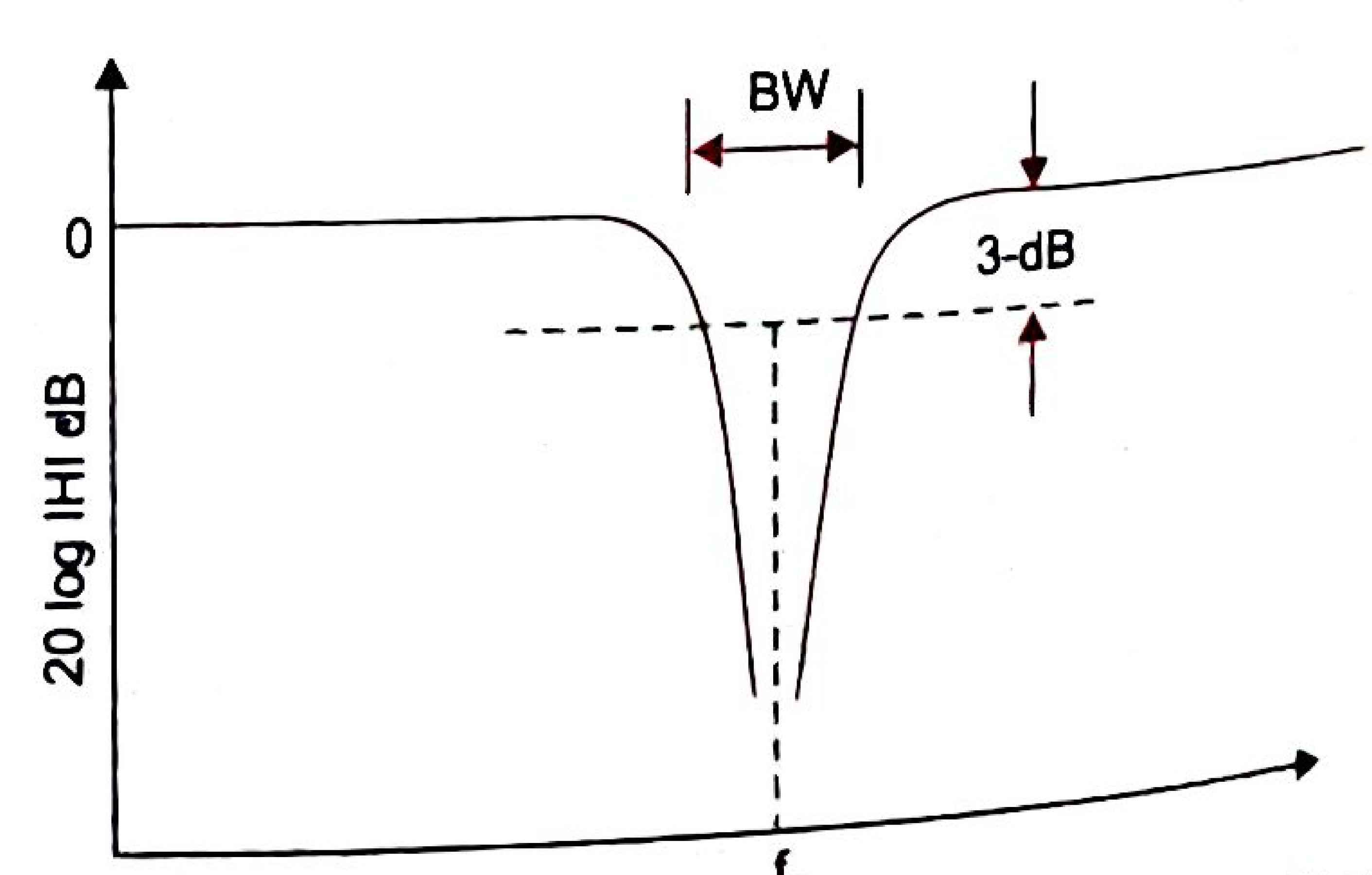


Fig. 7.18 (b) Frequency response of notch filter

Example 7.8

Design a 50 Hz active notch filter.

Solution

Given f_o = 50 Hz. Let C = 0.1 μF then from Eq. (7.62), we get $R = 1/2\pi f_o C$ = 1/2 (3.14) (50) (10^{-7}) = 31.8 kΩ.

For $R/2$, take two resistors of 31.8 kΩ in parallel and for $2C$, take two 0.1 μF capacitors in parallel to make the twin-T notch filter as shown in Fig. 7.15 (a) where resistors R_1 and R_2 are for adjustment of gain.

Wide Band-Reject Filter

A wide band-reject filter ($Q < 10$) can be made using a LPF, HPF and a summer. It is of course necessary that (i) the lower cut off-frequency f_l of the HPF should be much greater than the upper cut-off frequency f_h of the LPF and (ii) the pass band gain of LPF and HPF should be same.

Example 7.9

Design a wide band reject filter having f_h = 400 Hz and f_l = 2 kHz having pass band gain as 2.

Solution

For HPF, f_l = 2 kHz = $1/2\,\pi R_2 C_2$. Letting C_2 = 0.1 μF gives R_2 = 795 Ω (≈ 800 Ω). Again $A_o = A_{o2} = 2 = (1 + R_F/R_i)$ gives $R_F = R_i$ = 10 kΩ (say). For LPF, f_h = 400 Hz = $1/2\,\pi\, R_1 C_1$. Letting C_1 = 0.1 μF gives R_1 = 3978 Ω (choose 4 kΩ). Further $A_o = A_{o1} = 2 = (1 + R_F/R_i)$ gives $R_i = R_F$ = 10 kΩ (say). The schematic arrangement and the frequency response is shown in Figs. 7.16 (a, b).

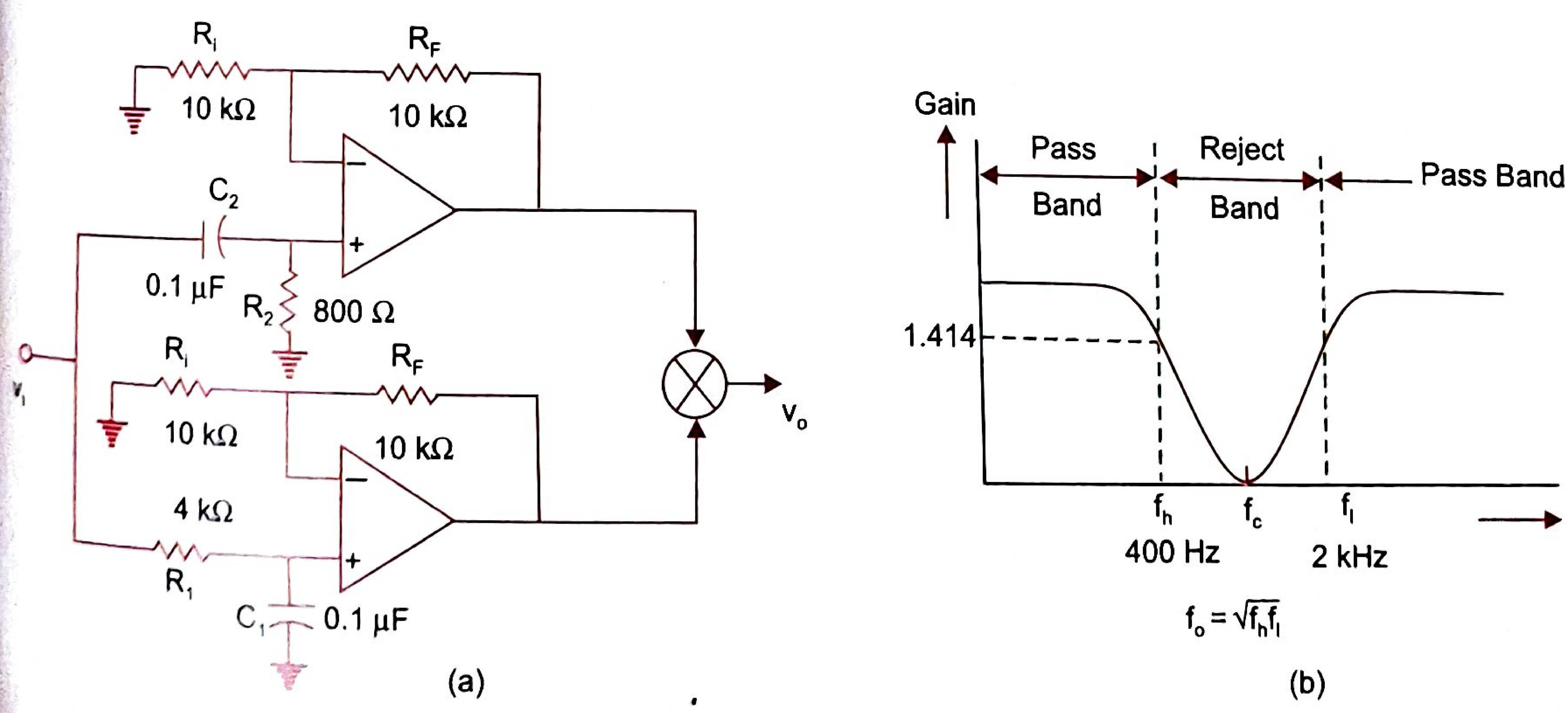


Fig. 7.19 (a) Wide band-reject filter (b) Frequency response

7.2.7 All Pass Filter

An all-pass filter passes all frequency components of the input signal without any attenuation and provides desired phase shifts at different frequencies of the input signal. When singals are transmitted over transmission lines, such as telephone wires, they undergo change in phase. These phase changes can be compensated by all-pass filters. Thus, all pass filters are also called delay equalizers or phase correctors. Figure 7.20 (a) shows an all-pass filter where $R_F = R_1$. The output voltage, v_o is obtained by using the superposition theorem:

$$v_o = \frac{-R_F}{R_1}v_i + \left(1+\frac{R_F}{R_1}\right)v_a \tag{7.77}$$

where v_a is the voltage at node 'A'.

Since $R_F = R_1$, Eq. (7.77) may be written as

$$v_o = -v_i + 2v_a \tag{7.78}$$

where, $$v_a = \frac{-jX_C}{R - jX_C} \times v_i \tag{7.79}$$

Putting the value of 'v_a' from Eq. (7.79) to Eq. (7.78), we get,

$$v_o = -v_i + 2.\frac{-jX_C}{R - jX_C}v_i \tag{7.80}$$

$$= v_i\left(-1 - \frac{2}{1 + j2\pi f RC}\right) \tag{7.81}$$

or $$\frac{v_o}{v_i} = \left(\frac{1 - j2\pi f RC}{1 + j2\pi f RC}\right) \tag{7.82}$$

The magnitude of $\frac{v_o}{v_i}$ is given by

$$\frac{|v_o|}{|v_i|} = \frac{\sqrt{1+(2\pi fRC)^2}}{\sqrt{1+(2\pi fRC)^2}} \tag{7.83}$$

It can be seen that $|v_o| = |v_i|$ throughout the frequency range. The phase shift ϕ between v_o and v_i is given by

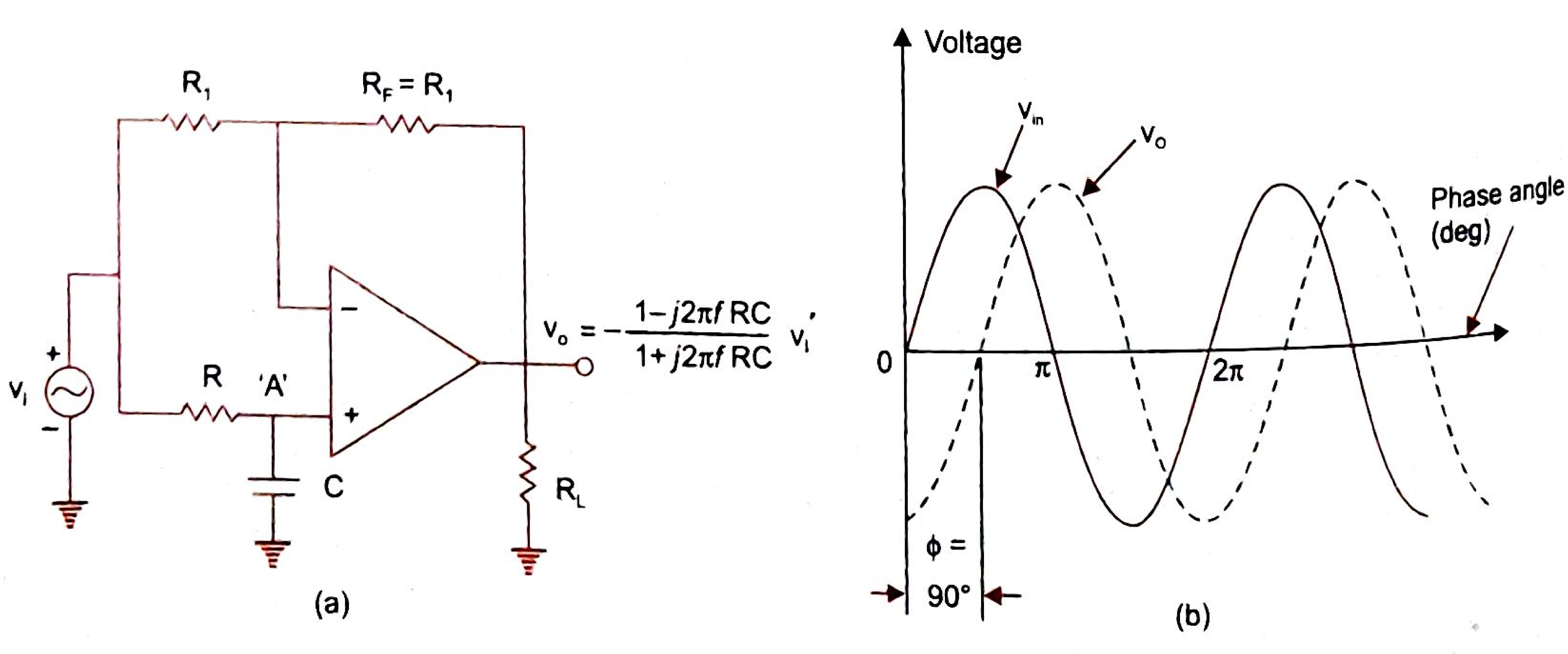


Fig. 7.20 (a) An all-pass filter (b) Input-output waveform for $\phi = 90°$

$$\phi = -\tan^{-1} 2\pi f RC - \tan^{-1} 2\pi f RC \quad (7.84\ (a))$$

$$= -2 \tan^{-1}(2\pi f RC) \quad (7.84\ (b))$$

Thus, the phase shift ϕ can be varied with frequency for a given R and C, and can be varied from 0 to –180° as the frequency is varied from 0 to ∞. As phase shift obtained is negative, the output v_o lags v_{in}. The phase shift can be made positive by interchanging R and C in Fig 7.20 (a).

If $R_F = R_1 = 10\ k\Omega$; $R = 15.9\ k\Omega$ and $C = 0.01\ \mu F$, using Eq. (7.75(b)), it can be seen that $\phi = -90°$. The output voltage v_o will have the same frequency as the input, but lags v_i by 90° as shown in Fig. 7.20 (b).

7.3 TRANSFORMATION

We shall now show that a high-pass, band-pass or band-reject filter can be obtained using an ideal low-pass transfer function by simple frequency transformation. For simplicity, let us normalize the frequency such that the cut-off frequency of the low pass function is unity. Let 'p' be the frequency domain of the low pass and 's' the frequency domain of interest.

Low Pass to High Pass Transformation

We get the high-pass characteristics by the following low-pass to high-pass transformation $p = 1/s$. For example, a third order Butterworth low-pass transfer function in p-domain given as

$$H(p) = \frac{A_o}{p^3 + 2p^2 + 2p + 1} \quad (7.85)$$

can be transformed to high-pass by the transformation $p = 1/s$ as

$$H(s) = \frac{A_o s^3}{s^3 + 2s^2 + 2s + 1} \quad (7.86)$$

The high-pass filter has the same pass band flatness as that of the Butterworth low-pass filter.

A low-pass filter can be transformed to a high-pass filter simply by interchanging R and C components and vice versa. A simple $RC : CR$ transformation is shown in Fig. 7.4 and Fig. 7.10. This is to note that the 3-dB (cut-off) frequency is the same for both the original low-pass and the transformed high-pass filter i.e., $f_{3\text{-dB}} = 1/2\pi RC$.

Low Pass to Band Pass Transformation

Consider a first order Butterworth low-pass transfer function in p-domain as

$$H(p) = \frac{A_o}{p + 1} \quad (7.87)$$

Let the transformation

$$p = \frac{s^2 + \omega_o^2}{(\omega_h - \omega_l)s} \quad (7.88)$$

In order to normalize, put

$$s_n = s/\omega_o \tag{7.89}$$

and quality factor, $Q = \dfrac{\omega_o}{\omega_h - \omega_l}$

Then Eq. (7.88) can be rewritten as

$$p = \frac{Q(s_n^2 + 1)}{s_n} \tag{7.90}$$

Substituting the transformation from Eq. (7.90) to Eq. (7.87), we get

$$H(s_n) = \frac{(A_o/Q)s_n}{s_n^2 + (1/Q)s_n + 1} \tag{7.91}$$

This is identical with Eq. (7.57) of band pass filter. The quality factor Q is an important parameter. If Q is very high, i.e. $Q >> 1$, the filter is called narrow band filter (i.e., $\omega_o >> (\omega_h - \omega_l)$) and the response is symmetric about the central frequency ω_o.

Low Pass to Band Reject Transformation

The transformation is given by

$$p = \frac{(\omega_h - \omega_l)s}{s^2 + \omega_o^2} = \frac{s_n}{Q(s_n^2 + 1)} \tag{7.92}$$

where $s_n = s/\omega_o$

The band-reject transfer function corresponding to first order low-pass of Eq. (7.70) and is given by

$$H(s_n) = \frac{A_o(s_n^2 + 1)}{s_n^2 + (1/Q)s_n + 1} \tag{7.93}$$

Note at $s_n = j1$, $|H(j1)| = 0$. Such filters are called 'notch filter' with normalized null frequency as $\omega_o = 1$.

7.4 STATE VARIABLE FILTER

The state variable configuration uses two op-amp integrators and one op-amp adder to provide simultaneous second order low-pass, band-pass and high-pass filter responses. The circuit can be viewed as analog computer simulation of biquadratic transfer function. Although, in general, all component values are different, imposing equal value simplifies algebra without diminishing versatility.

A simple state variable configuration has been shown in Fig. 7.21 (a). It uses two op-amp integrators and one op-amp summer. The outputs v_{HP}, v_{BP}, v_{LP} of high-pass, band-pass and low-pass filters are obtained at the output of op-amp A_1, A_2 and A_3 respectively. For simplification, it is assumed that V is the Laplace transform of the corresponding v in time domain.

The op-amp A_2 works as an inverting integrator, so the Laplace transformed output V_{BP} is given by

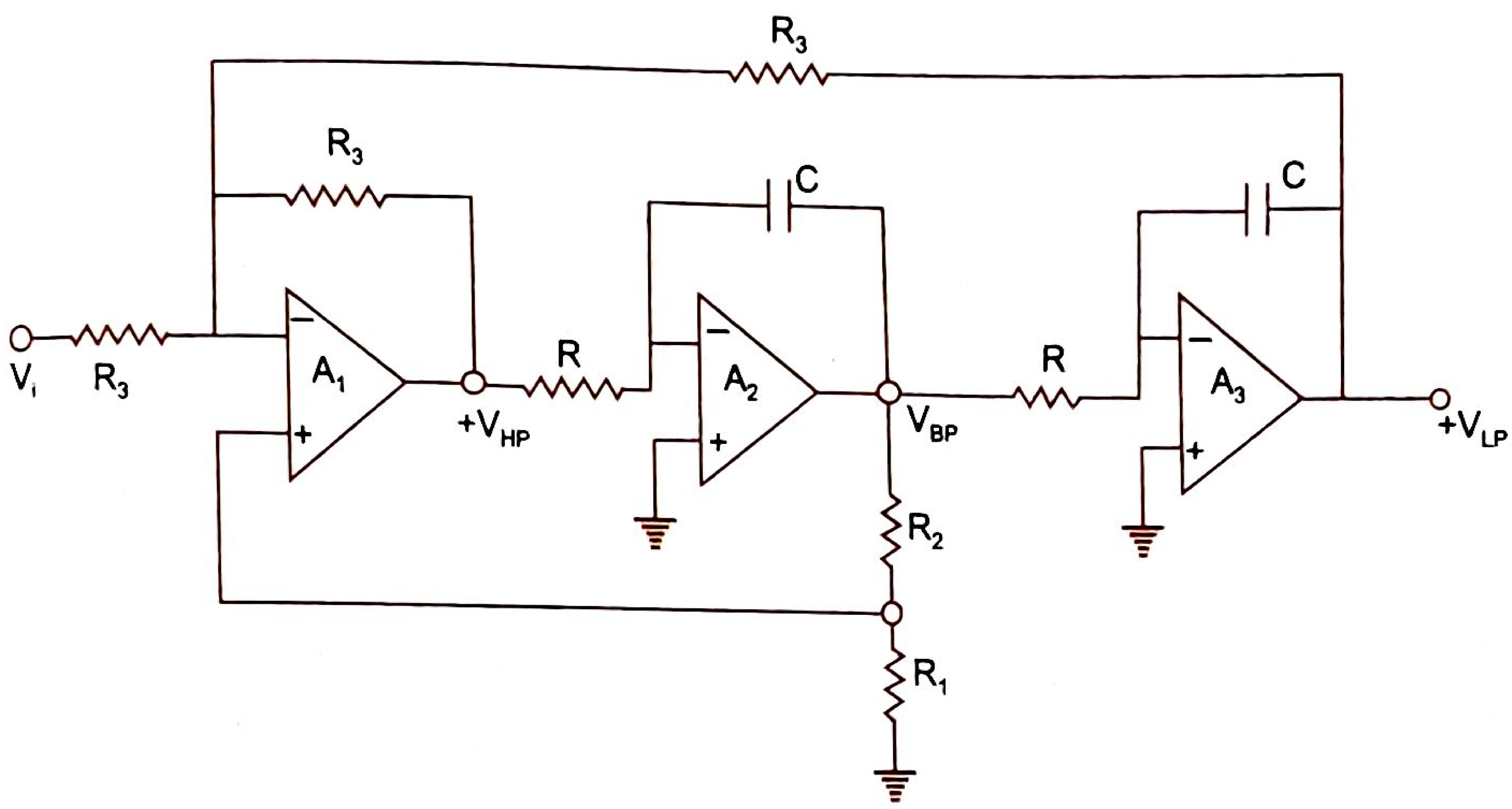


Fig. 7.21 (a) State variable filter

$$V_{BP} = -\frac{1}{RCs} V_{HP} \tag{7.94}$$

If $R = 1$ MΩ and $C = 1$ μF, so that $RC = 1$, we get,

$$V_{BP} = -\frac{1}{s} V_{HP} \tag{7.95}$$

Also for the inverting integrator A_3, we may write

$$V_{LP} = \frac{1}{s} V_{BP} = \frac{1}{s^2} V_{HP} \tag{7.96}$$

Op-amp A_1 is a three input summer. The output V_{HP} can be written using superposition theorem. That is,

$$V_{HP} = -\left(\frac{R_3}{R_3}\right)V_i - \left(\frac{R_3}{R_3}\right)V_{LP} + \left(1 + \frac{R_3}{R_3 || R_3}\right)\left(\frac{R_1}{R_1 + R_2}\right)V_{BP}$$

$$= -V_i - V_{LP} + 3\left(\frac{R_1}{R_1 + R_2}\right)V_{BP}$$

Put $$\alpha = 3\left(\frac{R_1}{R_1 + R_2}\right)$$

Then $$V_{HP} = -V_i - V_{LP} + \alpha V_{BP} \tag{7.97}$$

Eliminating V_{BP} and V_{LP} using Eqs. (7.95) and (7.96), we get

$$V_{HP} = -V_i - \frac{V_{HP}}{s^2} - \frac{\alpha}{s} V_{HP}$$

$$V_{HP}\left(1 + \frac{\alpha}{s} + \frac{1}{s^2}\right) = -V_i$$

So, the high pass transfer function H_{HP} is

$$H_{HP} = \frac{V_{HP}}{V_i} = \frac{-s^2}{s^2 + \alpha s + 1} \tag{7.98}$$

The damping factor α can be set by R_1 and R_2 for Bessel, Butterworth or Chebyshev response. Compare Eq. (7.98) to the standard high-pass transfer function of Eq. (7.36) as

$$\frac{A_o s^2}{s^2 + \alpha\omega_l s + \omega_l^2} \tag{7.99}$$

So for the high-pass filter of the state variable filter,

$$A_o = -1 \text{ and } \omega_l = 1$$

The low-pass transfer function is obtained by eliminating V_{HP} and V_{BP} from Eq. (7.97) as

$$H_{LP} = \frac{V_{LP}}{V_i} = \frac{-1}{s^2 + \alpha s + 1} \tag{7.100}$$

As in High-pass filter, the low-pass filter has

$$A_o = -1,\ \omega_h = 1$$

and $$\alpha = 3\left(\frac{R_1}{R_1 + R_2}\right)$$

Finally the band-pass impulse response is obtained from Eq. (7.97) by eliminating V_{HP} and V_{LP} as

$$H_{BP} = \frac{V_{BP}}{V_i} = \frac{s}{s^2 + \alpha s + 1} \tag{7.101}$$

The standard band-pass transfer function as given in Eq. (7.57) is

$$\frac{A_o \alpha \omega_o s}{s^2 + \alpha\,\omega_o s + \omega_o^2} \tag{7.102}$$

Comparing Eqs. (7.101) and (7.102), we get

$$A_o\ \alpha\ \omega_o = 1 \tag{7.103}$$

$$\omega_o = \frac{1}{RC} = 1, \text{ (}RC\text{ has been assumed to be 1)} \tag{7.104}$$

therefore, $$A_o = \frac{1}{\alpha} = \frac{R_1 + R_2}{3R_1} \tag{7.105}$$

From the analysis, we found that the band-pass response can be generated by integrating the high-pass response and that the low-pass is generated by integrating the band-pass.

The circuit of Fig. 7.21 (a) can be modified to that of Fig. 7.21 (b) where a fourth op-amp has been used to get a notch filter response. The op-amp A_4 provides the notch filter response by combining the low-pass and high-pass output. The notch filter output V_N is written as

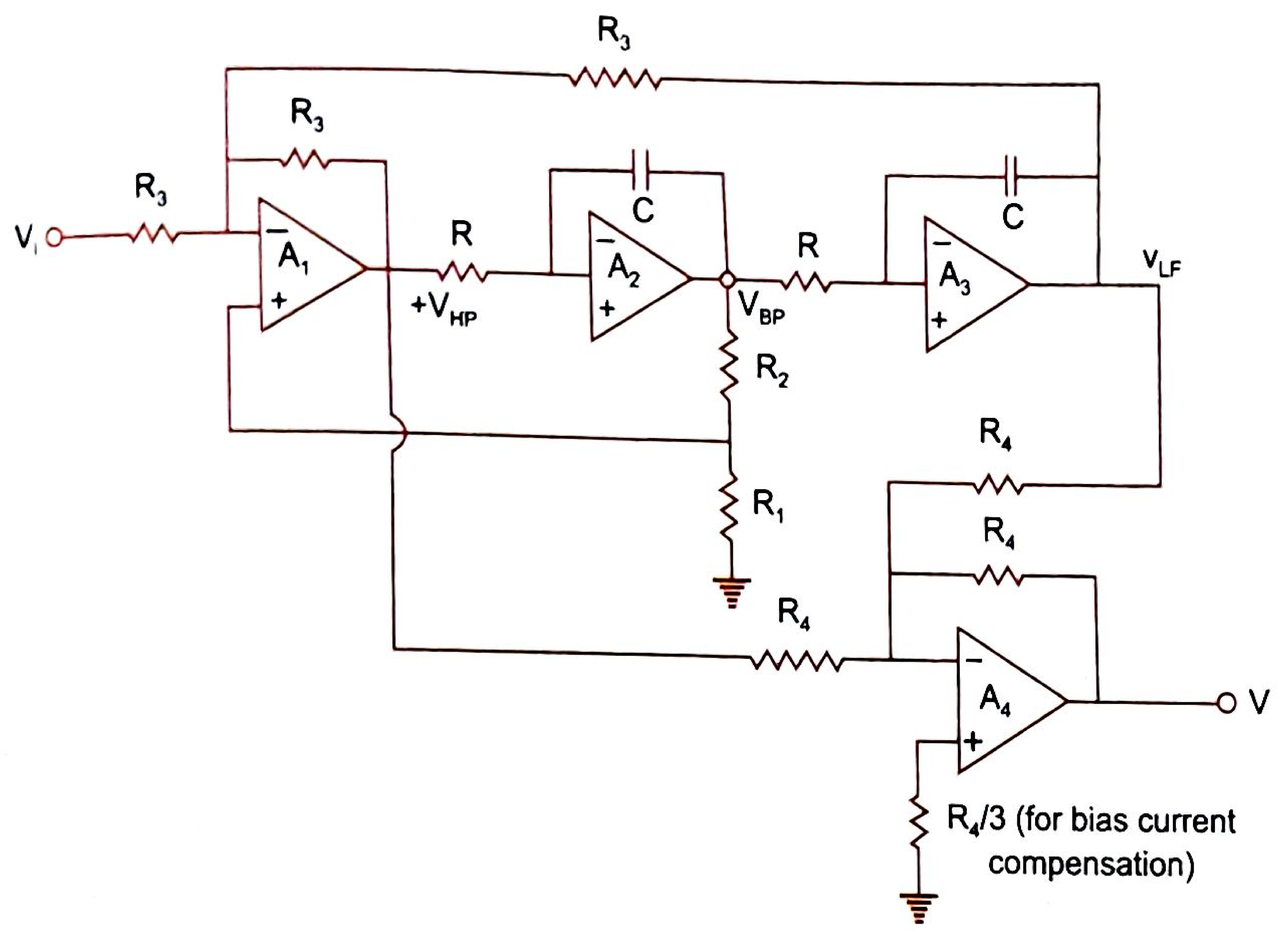


Fig. 7.21 (b) Four op-amp state variable filter with notch response

$$V_N = -\left(\frac{R_4}{R_4}\right)V_{HP} - \left(\frac{R_4}{R_4}\right)V_{LP} = -V_{HP} - V_{LP} \tag{7.106}$$

Putting the values of V_{HP} and V_{LP}, the transfer function of notch filter is obtained as

$$H_N = \frac{V_N}{V_i} = \frac{s^2+1}{s^2+\alpha s+1} \tag{7.107}$$

Thus it is possible to obtain LP, BP, HP and notch filter outputs from a state variable filter and therefore these are also known as universal filters. Quad op-amps such as LF347, TL074 and TLC274, FET input device are especially suited for these applications. With the advancement of IC technology, universal filters are available in single IC chip form. Datel's FLT-U2 and AF100 of National Semiconductor are the typical examples of IC universal filters.

State Variable Formulation

It may be noted that this filter is called state variable filter because the analog simulation can be made after the state variable formulation of the proper transfer function.

The band-pass filter transfer function of Eq. (7.57) can be rewritten for $A_o = -1$, $\omega_o = 1$, $\alpha = 1$ and assuming a dummy variable $X_1 = \mathcal{L}x_1(t)$, as

$$\frac{V_{BP}}{X_1} \cdot \frac{X_1}{V_i} = s\frac{1}{s^2+s+1} \tag{7.108}$$

Let

$$\frac{X_1}{V_i} = \frac{1}{s^2+s+1} \tag{7.109}$$

which can written in time domain as

$$\ddot{x}_1 + \dot{x}_1 + x_1 = v_i$$

Let
$$\dot{x}_1 = x_2$$

then
$$\dot{x}_2 = -x_1 - x_2 + v_i$$

These can be written in matrix form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_i \tag{7.110}$$

and
$$\frac{V_{BP}}{X_1} = s \tag{7.111}$$

leads to the output in time domain as $V_{BP} = \dot{x}_1 = x_2$

or,
$$V_{BP} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{7.112}$$

Similarly the high-pass transfer function H_{HP} of Eq. (7.98) with $\alpha = 1$ can be rewritten assuming another dummy variable $Y = \mathcal{L}y\ (t)$, as

$$H_{HP} = \frac{V_{HP}}{V_i} = -1 + \frac{s+1}{s^2+s+1} = -1 + \frac{Y}{V_i} \tag{7.113}$$

where
$$\frac{Y}{V_i} = \frac{Y}{X_1} \cdot \frac{X_1}{V_i} = (s+1) \cdot \frac{1}{s^2+s+1} \tag{7.114}$$

Let
$$\frac{X_1}{V_i} = \frac{1}{s^2+s+1} \tag{7.115}$$

It leads to

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_i \tag{7.116}$$

and
$$\frac{Y}{X_1} = (s + 1) \tag{7.117}$$

leads to in time domain $y = \dot{x}_1 + x_1 = x_2 + x_1$

or,
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{7.118}$$

Hence the output

$$V_{HP} = y - v_i = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - (1)\, v_i \tag{7.119}$$

The simulation of Eqs. (7.116) and (7.119) is shown in Fig. 7.21 having V_{HP} as output. Similarly, the low-pass transfer function of Eq. (7.84) can be written in the same fashion as

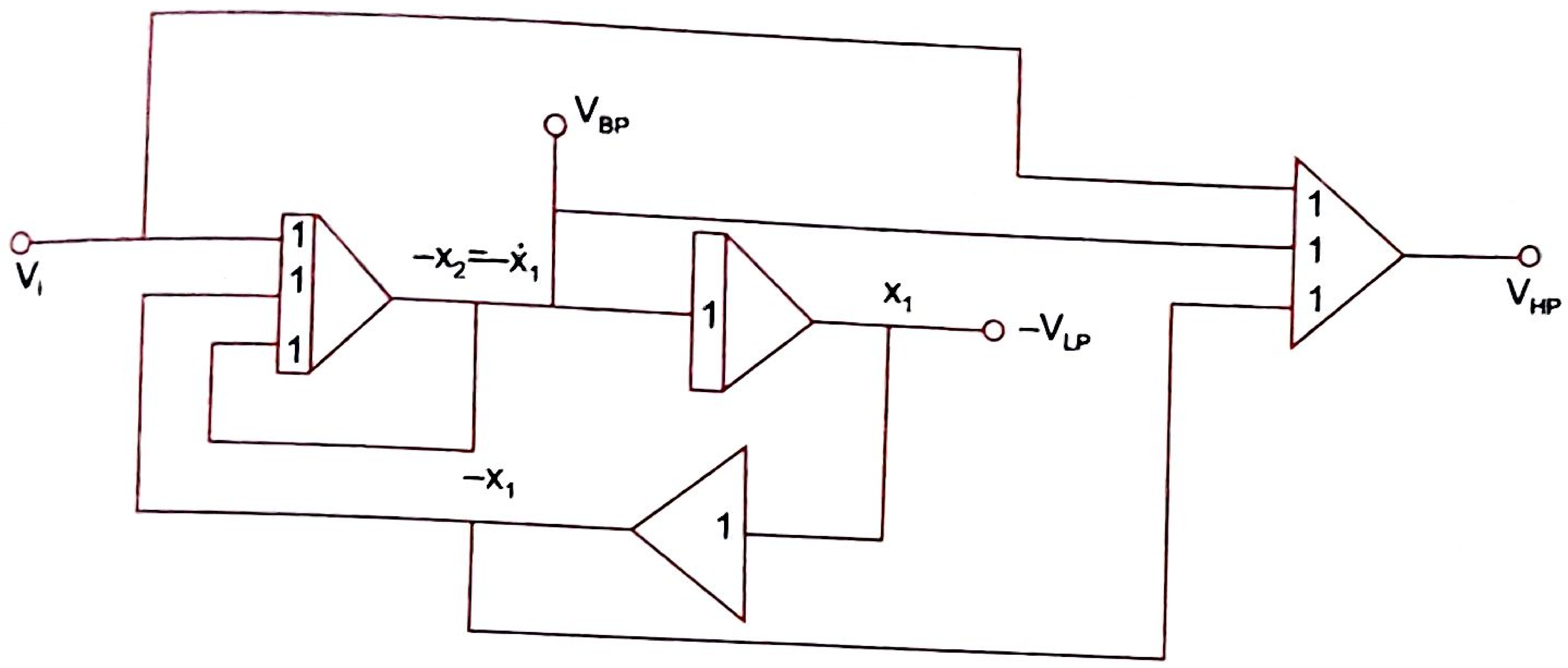


Fig. 7.22 Simulation of state variable filter

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_i \tag{7.120}$$

and output

$$V_{LP} = \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The simulation is as shown in Fig. 7.22.